INVESTMENT DECISIONS BASED ON UNCERTAINTY ANALYSIS AND MULTISEQUENTIAL APPROACH OF EFFICIENCY

Carmen TURLEA[1]
Ioana GHIDIU [2]

ABSTRACT
The paper approaches the problem of making sequential decisions in allocation critical resourses for undertaking an investment projects (developed on a long period of time – several years). The investment decisions are based on the performance indicators measuring the impacts, outcomes, outputs and inputs of the project. These indicators give during the life cycle of the project indications on allocation various resources in the implementation stage and register the progress toward project objectives. The relevance of the proposed subject relates to the problem of making estimations on the benefits and costs at each years of the forecasted horizon and it involves the dynamic programming method (inspired by the works of R. Bellman) to minimize the standard deviation of the net present value for the investment project. The indicators used in investment appraisal theory such as the NPV and IRR are approached as uncertain variables and the dynamic programming method is applied as a way minimize the expected variability in project outcomes. The newly suggested way to interpret the uncertainty that is associated inherently to the DCF indicators is meant to enhance the ability to make long-term investment decision on marginal projects.

KEYWORDS: dynamic programming, investment decision, net present value, standard deviation

JEL CLASSIFICATION: *G17, G31*

1. INTRODUCTION

An important function of the investment appraisal concerns uncertainty on the influencing factors on the performance measure which consists of a specific form of net revenue or profit. Projecting discounted cash flows over a period of time requires assigning specific values to inflation rates, future regulatory costs and other factors that, in reality, remain uncertain. The investment appraisal includes in the computation of the performance indicators the variables corresponding to the various categories of benefits and expenditures – this is dome by assigning probabilities to ranges of outcomes. This approach brings a realistic view on the model computations and calls for including in the model even sensitivity analysis and risk assessment. Usually, this is done by probability analysis as a way to expressing indetermination in the building the financial model. Along this technique based on probability, the fuzzy logic can be also used to represent subjective belief, the uncertainty and vagueness in the real world.
Constructing plausible measures of the costs and benefits of specific actions is often an intricate task. In practice, analysts try to estimate costs and benefits either by using survey methods or by drawing inferences from market behavior.
Some critical aspects of applying the DCF method on a multi-period time horizon deal with the following aspects:

[1] The Bucharest University of Economic Studies, Romania, e-mail: turlea_carmen@yahoo.ro
[2] The Bucharest University of Economic Studies, Romania

• The time factor affects the perceived worth of the inflows and the outflows during the time span of the project life. According to the future prospects of the cash flow dynamics, a discount factor is used ranging between 5 and 20% (the most frequently used rates). There is no commonly agreed discount rate for an investment project because contextual factors intervene in choosing it on the specific situations.
• Measurement of costs and benefits: sometimes monetary values have to be estimated in the partial lack of information on the sales, the cost for inputs, of market prices, etc. thus, the estimations may need to be adjusted as the time passes to reflect the actual values of sales of goods, production costs, operational expenditures etc.
The term "*risk and uncertainty*" tends to be applied generically to the analysis of situations with unknown outcomes. Usually, in investment appraisal the conventional distinction between risk and uncertainty made in the library literature is used. In essence, risk is a quantity subject to empirical measurement, while uncertainty is of a non-quantifiable type. Thus, in a risk situation it is possible to indicate the likelihood of the realized value of a variable falling within stated limits - typically described by the fluctuations around the average of a probability calculus. On the other hand, in situations of uncertainty, the fluctuations of a variable are such that they cannot be described by a probability calculus. Thus, risk and uncertainty are best thought of as representing a spectrum of unknown situations with which an analyst may be dealing, ranging from perfect knowledge of the likelihood of all the possible outcomes at one end (i.e., risk) to no knowledge of the likelihood of possible outcomes at the other (i.e., uncertainty).

2. THE INVESTMENT DECISION BASED ON DISCOUNTING CASH FLOWS

The investment decisions in project appraisal involves resources; allocation and may take much efforts in developing correct technical forms for a sound basis in making rational use of scarce resources when alternative uses are possible. The purpose of investment appraisal is to assess the economic prospects of a proposed investment project; usually it computes indicators such as net present value, the internal rate of return, the payback period, profit before the tax, profit after the tax etc. all of them being subject to indetermination. Along with these, in practice, several more measures of financial return interest the analyst because they are simpler (e.g. payback) or because of personal preferences, or because different measures provide extra information (e.g. payback provides a crude indication of liquidity, return on investment (*ROI*) provides some indication of the project impact on reported profit).
Various studies indicate that the outcomes of the investment appraisal analysis should be treated with caution because they may be highly inaccurate. There is an inherent risk in making the financial projections, likely to lead to inefficient decisions, as defined by Pareto and Kaldor-Hicks efficiency (Flyvbjerg, Bruzelius and Rothengatter, 2003). These outcomes (almost always tending to underestimation unless significant new approaches are overlooked) are to be expected because such estimates:

- Rely heavily on past like projects (often differing markedly in function or size and certainly in the skill levels of the team members)
- Rely heavily on the project's members to identify (*remember* from their collective past experiences) the significant cost drivers
- Rely on very crude heuristics to estimate the money cost of the intangible elements.
- Are unable to completely dispel the usually unconscious biases of the team members (who often have a vested interest in a decision to go ahead) and the natural psychological tendency to *"think positive"* (whatever that involves).

In recent years, several authors have studied the evaluation of the expected net present value and the variance of the net present value of probabilistic cash flows under random timings (Kahraman, 2001). Buck and Askin (1986) have defined such related risks measures, Schlaifer (1961) and Morris (1968) developed Gaussian linear loss integrals as measures over a partial domain of X in contrast to full domain measures given by traditional statistical moments. Also, Hillier in 1963 introduced an analytical method which determines the probability distribution function of the net present value and the internal rate of return of a series of random discrete cash flows which occur at constant times.

The estimated free cash flow (*CF*) is the sum of the real income and expenditures that occur during a chosen time unit. The CF is the simple difference between cash receipts and cash expenditures over the entire life of the project. When investing in a project, any investor or an interested part will take into consideration the decreasing worth of inflow of money in the project's budget and, therefore, a risk factor proportional to interest rate is necessary to relate to this issue (called the risk-adjusted discount rate).

The *net present value (NPV*) of a project is equal the sum of the present value of the cash-flows over different periods associated with the project. The net present value represents the net benefit over and above the compensation for time and risk.

$$NPV = \sum_{h=1}^{T} \frac{CF_t}{(1+r)^h} - \sum_{h=1}^{D} \frac{I_t}{(1+r)^h} \qquad (1)$$

Where the *T* describes the expected project life (number of years for the project), the *D* designates the investment cycle (number of years for investments); the *r* is the risk-adjusted discount factor.

The basic decision rule for a project appraisal is simply to accept depending on the positive value for NPV. Usually, the net present value (*NPV*) is computed as an indication of the project's contribution to the company's shareholder value. The equation (1) reflects the idea that the free cash flows of the relevant periods are readjusted to the risk-adjusted discount rate at the start of the project. As it was mentioned before, the traditional decision rule stipulates that if the *NPV* is positive (regardless the value) the project should be undertaken. Alternatively, the same rule is similar to $IRR > r^*$ when comparing the internal rate of return (IRR) to the marginal cost of the capital of a specific company:

$$\sum_{h=1}^{T} \frac{CF_t}{(1+IRR)^h} = \sum_{h=1}^{D} \frac{I_t}{(1+IRR)^h} \qquad (2)$$

A project whose single-value NPV is small may still be accepted following risk analysis, on the grounds that its overall chances for yielding satisfactory return are greater than is the probability of making an unacceptable loss. Likewise, a marginally positive project could be rejected on the basis of being excessively risky, or one with a lower *NPV* may be preferred to another with a higher *NPV* because of a better risk/return profile. It bridges the communication gap between the analyst and the decision maker. Thus, the final decision is subjective: whether to change projects with a relatively high return showing less concern about the risk involved or to change the project with relatively modest but safe returns. The analyst should take due care to identify the major correlated variables and to adequately provide for the impact of such correlations in the simulation. Finally, overlooking significant inter-relationships among the projected variables can distort the results of risk analysis and may lead to misleading conclusions.

The newly suggested approach consists in involving the dynamic programming methodology to minimize the risks associated with the uncertain variables expressed by the projects' benefits and costs. The basic principle to be followed is that one of the optimality stating that "*an optimal policy has the property that whatever the initial state and initial decision are, the remaining decision must constitute an optimal policy with regard to the state resulting from the first decision*" (formulated originally by Bellman, cited form Hillier and Lieberman 2005).

The used *risk variable* is the standard deviation in the CF variables over time, defined as one which is critical to the viability of the project in the sense that a small deviation from its expected value is both probable and potentially damaging to the project worth.

As it is seen, the expected value and the variance of a probabilistic cash flow (*CF*) are obtained by means of moments. The project standard deviation is obtained from the variance of the cash flows on the entire time horizon as:

$$\sigma_P^2 = \sum_{t=1}^{T} \frac{\sigma_{CFt}^2}{(1+r)^{2\cdot t}} \tag{3}$$

where σ_{CF_t} is the standard deviation of the cash flow estimation in the t stage.

$$\sigma_{CF_t} = \sum_{i=1}^{p} \frac{(CF_{t,i} - E[CF_t])^2}{p} \tag{4}$$

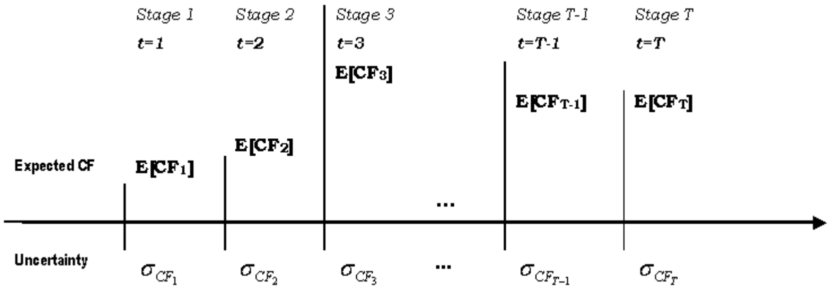

Figure 1. The stream of cash flows at different states

In the allocating decision at one specific stage *t, t=1,...,T,* there are several states that are possible to occur, consequently a certain value for the benefits and the cost appear – making possible the computations in the cash flows corresponding to the *t* stage: $CF_{t,1}$, $CF_{t,2}$, … $CF_{t,p}$; and $E[CF_t]$ is the expected value of the cash flow stream at the *t* stage.

3. THE DYNAMIC PROGRAMMING METHOD USED TO REDUCE UNCERTAINTY ON ESTIMATIONS

Dynamic programming is a technique of decision making in a sequence of interrelated decisions. Each stage has a number of states (either finite or infinite). The effect of the current decision at each stage is to transform the current state to a state at the next stage (the analyst's decision of designated the next destination (in terms of a specific estimation for benefits and cost for the next period) led it to a state at the next stage).

Respecting the general feature of the dynamic programming approach, the solution procedure is designed to find an optimal policy for the overall problem, i.e. a prescription of the optimal policy decision at each stage for each of the possible states. Together with the optimal policy, the optimal value of the objective is found at each stage for each of the possible states. The solution procedure starts at the end and moves backward stage by stage until it finds the optimal policy starting at the initial stage. The backward procedure begins by finding the optimal policy for the last stage, which is usually quite simple.

Let X_t designates the decision variable in the *t* stage – the chosen estimation of the cash flows at *t* moment of time – meaning $E[CF_t]$.

The $f_t(s, X_t)$ is the uncertainty estimation function for the cumulated cash flows on the remaining stages: *t+1, t+2,…T*; *s* is the state of nature imminent to occur, the programming of the allocation for the projects' resources starts at the *t* stage. Let X_t^* denotes the corresponding minimum value of the $f_t(s, X_n)$ function:

$$f_t^*(s) = \min_{X_t} f_t(s, X_t) = f_t(s, X_t^*) \tag{5}$$

A recursive relationship that identifies the optimal policy for stage *t*, given the optimal policy for stage *t+1*, is available:

$$f_t(s, X_t) = f_{t+1}(X_{t+1}) + f_t(X_t, X_{t+1}^*) = f_{t+1}(X_{t+1}) + \min_{X_{t+1}} \sigma^2(X_t, X_{t+1}) \tag{6}$$

meaning that the overall uncertainty at t stage equals the immediate uncertainty (at *t+1* stage) plus the minimum future value of the uncertainty function (on *t+1* stages onward).
Both the minimum value of the uncertainty function $f_t^*(X_t)$ and the optimal policy X_{t+1}^* are defined. In the backward procedure, the objective is to find $f_1^*(X_t)$ and the corresponding route (having in mind the algorithm of the shortest path). Dynamic programming finds it by successively finding: $f_{T-1}^*(s, X_t)$, $f_{T-2}^*(s, X_t)$, … for each of the possible *s* states and then using $f_2^*(s, X_t)$ to solve for $f_1^*(X_t)$.

4. CONCLUSIONS

Recognizing the fact that the envisaged values for the benefits and expenditures in the project undertaking are subject to uncertainty, the investment appraisal indicators should be supplemented with additional uncertainty techniques: sensitivity and scenario analysis tests.

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