HARMONY SEARCH ALGORITHM FOR SOLVING TWO AGGREGATE PRODUCTION PLANNING MODELS WITH BREAKDOWNS AND MAINTENANCE

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ABSTRACT
Aggregate Production planning (APP) and preventive maintenance (PM) are most important issue carried out in manufacturing environments which seeks efficient planning, scheduling and coordination of all production activities that optimizes the company's objectives. In this paper, we develop two mixed integer linear programming (MILP) models for an integrated aggregate production planning system with return products, breakdowns and preventive maintenance. The goal is to minimize production breakdowns and Preventive maintenance costs and instabilities in the work force, inventory levels and downtimes, also effect of PM on the objective function. Additionally, Taguchi method is conducted to calibrate the parameter of the meta-heuristic and select the optimal levels of the algorithm's performance influential factors. Due to NP-hard class of APP, we implement a harmony search (HS) algorithm for solving these models. Finally, computational results show that, the objective values obtained by APP with PM are better from APP with breakdowns results.

KEYWORDS: *Harmony search, Aggregate production planning, Preventive maintenance, Taguchi method*

JEL CLASSIFICATION: *D24*

1. INTRODUCTION

Aggregate production planning belongs to a class of production planning problems in which there is a single production variable representing the total production of all products (Dilworth, 1993). APP is a medium range capacity planning method that typically encompasses a time horizon anywhere from 2 to 18 months. The aims of APP are to set overall production levels for each product category to meet fluctuating or uncertain demand in the near future and to set decisions concerning hiring, layoffs, overtime, backorders, subcontracting, inventory level and determining appropriate resources to be used (Wang and Liang, 2004).
A survey of models and methodologies for APP has been represented by Nam and Ogendar (1992). Ashayeri et al. proposed a model optimizing total maintenance and production costs in discrete multi-machine environment with deterministic demand (Ashayeri et al., 1995). This paper proposes and discusses models to generate such integrated APP and maintenance plans which aim at achieving an optimal trade-off between the various production and maintenance costs. At the tactical level, there are only few papers discussing this issue. Wienstein and Chung presented a three-part model to resolve the conflicting objectives of system reliability and profit maximization. An aggregate production plan is first generated, and then a master production schedule is developed

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to minimize the weighted deviations from the specified aggregate production goals. Finally, work center loading requirements, determined through rough cut capacity planning, are used to simulate equipment failures during the aggregate planning horizon. Several experiments are used to test the significance of various factors for maintenance policy selection. These factors include the category of maintenance activity, maintenance activity frequency, failure significance, maintenance activity cost, and aggregate production policy (Wienstein & Chung, 1999). Sortrakul et al. proposed an integrated maintenance planning and production scheduling model for a single machine minimizing the total weighted expected completion time to find the optimal PM actions and job sequence (Sortrakul et al., 2005). Aghezzaf and Najid discuss the issue of integrating production planning and preventive maintenance in manufacturing production systems. In particular, it tackles the problem of integrating production and preventive maintenance in a system composed of parallel failure-prone production lines. It is assumed that when a production line fails, a minimal repair is carried out to restore it to an 'as-bad-as-old' status. Preventive maintenance is carried out, periodically at the discretion of the decision maker, to restore the production line to an 'as-good-as-new' status. It is also assumed that any maintenance action, performed on a production line in a given period, reduces the available production capacity on the line during that period (Aghezzaf & Najid, 2008). Nourelfath and Chatelet paper deals with the problem of integrating preventive maintenance and tactical production planning, for a production system composed of a set of parallel components, in the presence of economic dependence and common cause failures. Economic dependence means that performing maintenance on several components jointly costs less money and time than on each component separately. Common cause failures correspond to events that lead to simultaneous failure of multiple components due to a common cause (Nourelfath & Chatelet, 2012). Fitouhi and Nourelfath developed a model for planning production and noncyclical preventive maintenance simultaneously for a single machine, subjected to random failures and minimal repairs. The proposed model determines simultaneously the optimal production plan and the instants of preventive maintenance actions. The objective is to minimize the sum of preventive and corrective maintenance costs, setup costs, holding costs, backorder costs and production costs, while satisfying the demand for all products over the entire horizon. The problem is solved by comparing the results of several multi-product capacitated lot-sizing problems. The value of the integration and that of using noncyclical preventive maintenance when the demand varies from one period to another are illustrated through a numerical example and validated by a design of experiment. The later has shown that the integration of maintenance and production planning can reduce the total maintenance and production cost and the removal of periodicity constraint is directly affected by the demand fluctuation and can also reduce the total maintenance and production cost (Fitouhi & Nourelfath, 2012). Fitouhin and Nourelfath presented an integrated model for production and general preventive maintenance planning for multi-state systems. For the production side, the model generates, for each product and each production planning period, the quantity of inventory, backorder, items to produce and also the instant of set-up. For the maintenance side, for each component, we proposed the instant of each preventive maintenance action which can be carried out during the production planning period. A Matrix based methodology was used in order to estimate model parameters such as system availability and the general capacity. The proposed model was solved by the ES method and SA (Fitouhin & Nourelfath, 2014). The large part of the production planning models assumes that the system will function at its maximum performance during the planning horizon, and the large part of the maintenance planning models disregards the impact of maintenance on the production capacity and does not explicitly consider the production requirements (Aghezzaf et al., 2007). It is therefore crucial that both production and maintenance aspects related to a production system are concurrently considered during the elaboration of optimal production and maintenance plans. The purpose of this paper is to develop a combined production planning model for two phase production systems, breakdowns and preventive maintenance in an aggregate production planning. The main objective of the proposed models are to determine an

integrated production and maintenance plan that minimizes the expected total production and maintenance costs over a planning horizon and effect of preventive maintenance on the aggregate production planning model.
The remaining of this paper is organized as follows: Section 2 describes an aggregate production planning Model with machine breakdowns, and a MILP formulation of the aggregate production planning Model with preventive maintenance. The solution approach harmony search (HS) is presented in Section 3. Section 4 presents computational experiments. The conclusions and suggestions for future studies are included in Section 5.

2. **The mathematical models**

2.1. The APP model and breakdowns

In this section, we present an aggregate production planning model with machine breakdowns. This model is relevant to multi-period, multi-product, multi-machine, two-phase production systems.

2.1.1. Assumptions

- The quantity shortage at the beginning of the planning horizon is zero.
- The quantity shortage at the end of the planning horizon is zero.
- Breakdown decision variable, if setup to be performed, the decision variable is equal to one, and otherwise it is zero.
- There is a setup cost of producing a product only once at the beginning of a period, and the setup cost after a failure is not considered.

2.1.2. Model variables

P_{i2t}: Regular time production of second-phase product *i* in period *t* (units).
O_{i2t}: Over time production of second-phase product *i* in period *t* (units).
C_{i2t}: Subcontracting volume of second-phase product *i* in period *t* (units).
B_{i2t}: Backorder level of second-phase product *i* in period *t* (units).
I_{i2t}: The inventory of second-phase product *i* in period *t* (units).
H_t: The number of second group workers hired in period *t* (man-days).
L_t: The number of second group workers laid off in period *t* (man-days).
W_t: Second workforce level in period *t* (man-days).
Y_{i2t}: The setup decision variable of second-phase product *i* in period *t*, a binary integer variable.
XR_{i2t}: The number of second-phase returned products of product *i* that remanufactured in period *t*.
XRI_{i2t}: The number of second-phase returned products of product *i* held that in inventory at the end of period *t*.
XD_{i2t}: The number of second-phase returned products of product *i* that disposed in period *t*.
P_{k1t}: Regular time production of first-phase product *k* in period *t* (units).
O_{k1t}: Over time production of first-phase product *k* in period *t* (units).
C_{k1t}: Subcontracting volume of first-phase product *k* in period *t* (units).
B_{k1t}: Backorder level of first-phase product *k* in period *t* (units).
I_{k1t}: The inventory of first-phase product *k* in period *t* (units).
H'_t: The number of first group workers hired in period *t* (man-days).
L'_t: The number of first group workers laid off in period *t* (man-days).
W'_t: First workforce level in period *t* (man-days).
Y_{k1t}: The setup decision variable of first-phase product *k* in period *t*, a binary integer variable.

2.1.3. Parameters

p_{k1t}: Regular time production cost of first-phase product k in period t ($/units).
o_{k1t}: Over time production cost of first -phase product k in period t ($/units).
c_{k1t}: Subcontracting cost of first-phase product k in period t ($/units).
h_{k1t}: Inventory cost of first-phase product k in period t ($/units).
a_{k1l}: Hours of machine l per unit of first-phase product k (machine-days/unit).
u_{k1l}: The setup time for first-phase product k on machine l (hours).
r_{k1lt}: The setup cost of first-phase product k on machine l in period t ($/machine-hours).
R'_{kt}: The regular time capacity of machine l in period t (machine-hours).
hr'_t: Cost to hire one worker in period t for first group labor ($/man-days).
l'_t: Cost to layoff one worker of first group in period t ($/man-days).
w'_t: The first group labor cost in period t ($/man-days).
I_{k10}: The initial inventory level of first-phase product k in period t (units).
w'_0: The initial first group workforce level (man-days).
B_{k10}: The initial first group backorder level (man-days).
e_{k1}: Hours of labor per unit of first-phase product k (man-days/unit).
α'_t: The ratio of regular-time of first group workforce available for use in overtime in period t.
β'_{lt}: The ratio of regular time capacity of machine l available for use in overtime in period t.
$w'_{max\,t}$: Maximum level of first group labor available in period t (man-days).
D_{i2t}: Forecasted demand of second-phase product i in period t (units).
p_{i2t}: Regular time production cost of second-phase product i in period t ($/units).
o_{i2t}: Over time production cost of second-phase product i in period t ($/units).
c_{i2t}: Subcontracting cost of second-phase product i in period t ($/units).
h_{i2t}: Inventory cost of second-phase product i in period t ($/units).
a_{i2j}: Hours of machine j per unit of second-phase product i (machine-days/unit).
u_{i2j}: The setup time for second-phase product i on machine j (hours).
r_{i2jt}: The setup cost of second-phase product i on machine j in period t ($/machine-hours).
R_{jt}: The regular time capacity of machine j in period t (machine-hours).
hr_t: Cost to hire one worker in period t for second group labor ($/man-days).
l_t: Cost to layoff one worker of second group in period t ($/man-days).
w_t: The first group labor cost in period t ($/man-days).
I_{i20}: The initial inventory level of second-phase product i in period t (units).
w_0: The initial second group workforce level (man-days).
B_{i20}: The initial second group backorder level (man-days).
e_{i2}: Hours of labor per unit of second-phase product i (man-days/unit).
α_t: The ratio of regular-time of second group workforce available for use in overtime in period t.
β_{jt}: The ratio of regular time capacity of machine j available for use in overtime in period t.
f: The working hours of labor in each period (man-hour/man-day).
$w_{max\,t}$: Maximum level of second group labor available in period t (man-days).
$C_{max\,it}$: Maximum subcontracted volume available of second-phase product i in period t (units).
f_{ik}: The number of unit of first-phase product k required per unit of first-phase product i.
TR_{i2t}: The number of second-phase returned products of product i in period t.
$XD_{max\ i2t}$: The maximum number of second-phase returned products of product i that could be disposed in period t.
$XR_{max\ i2t}$: The maximum number of second-phase returned products of product i that could be remanufactured in period t.
hX_{i2t}: Inventory cost of second-phase returned products of product i in period t ($/units).
$C1_{l1t}$: Failure cost of first-phase machine l in period t ($).
$C3_{j2t}$: Failure cost of second-phase machine j in period t ($).
$C5_{i2t}$:The cost of returned products of second-phase product i that disposed in period t ($).

$C6_{i2t}$: The cost of returned products of second-phase product i that remanufactured in period t ($).
m : Percentage of machine capacity in each period (due to lack of maintenance in the previous period) is lost due to Failure.
LT :Lead time.
M: A large number.

2.1.4. First proposed Model

The first term in objective function (1) is total production cost, which is associated with the regular-time production, overtime production and subcontracting cost for the second-phase products. The second term in objective function (1) is total production cost, which is associated with the regular-time production, overtime production and subcontracting cost for the first-phase products. The third and fourth terms in (1) are inventory cost for the second-phase and first-phase products. The fifth and sixth terms in (1) are total setup cost for the second-phase and first-phase products. The seventh and eighth terms in (1) are backorder setup cost for the second-phase and first-phase products. The ninth and tenth terms in (1) are total labor cost and hiring and layoff cost associated with the change of workforce level for the second-phase. The eleventh and twelfth terms in (1) are total labor cost and hiring and layoff cost associated with the change of workforce level for the first-phase. The thirteenth term in (1) is failure cost for the first-phase. The fourteenth term in (1) is failure cost for the second-phase. The fifteenth term in (1) is disposed cost for the second-phase products. The sixteenth term in (1) is remanufactured cost for the second-phase products. The seventeenth term in (1) is inventory cost for the second-phase products.

Constraint (2) is relevant to satisfy demands for the second-phase products. Constraint (3) ensures production, subcontracting and inventory equilibrium for first-phase products that associated to the total production of second-phase products. Constraint (4) certifies that the initial inventory level and the subcontracting volume of first-phase products in the beginning of planning horizon should be equal or greater than the total production of second-phase products at the first LT periods to satisfy the products demand. Constraints (5) and (6) limit the regular time production to the available second group machines capacity and the overtime production to the available overtime for this group of machines respectively. Setup times are considered in the machine capacity constraint (5). Also, total production of first-phase products in each period of regular time and overtime is limited by the available production capacity for the first group machines by constraints (7) and (8) respectively. Constraints (9) and (10) are relevant the relationship between production and setup variables for the first-phase and second-phase products respectively. Constraints (11) and (12) are relevant to workforce level for the both groups of workers. Constraints (13)–(16) imply workforce capacity constraints at regular time and overtime at each period for the both groups of workers. Constraints (17) and (18) limit the workforce level to the available labor for the both groups of workers. Constraint (19) limits the subcontracting level to the available subcontracting volume. Naturally in order to minimizing the objective function, the constraints (20) and (21) are not necessary and we can ignore them. Constraint (22) is balance of return products. Constraint (23) limits the disposed level to the available disposed volume. Constraint (24) limits the remanufactured level to the available remanufactured volume. Constraints (25) and (26) are the setup decision variable for the both phase.

$$\begin{aligned} MinZ = &\sum_{i=1}^{N}\sum_{t=1}^{T}(p_{i2t}P_{i2t}+o_{i2t}O_{i2t}+c_{i2t}C_{i2t})+\sum_{k=1}^{K}\sum_{t=1}^{T}(p_{k1t}P_{k1t}+o_{k1t}O_{k1t}+c_{k1t}C_{k1t})+ \\ &\sum_{t=1}^{T}\sum_{i=1}^{N}h_{i2t}I_{i2t}+\sum_{t=1}^{T}\sum_{k=1}^{K}h_{k1t}I_{k1t}+\sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{J}r_{i2jt}Y_{i2t}+\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{l=1}^{L}r_{k1lt}Y_{k1t}+\sum_{i=1}^{N}\sum_{t=1}^{T}b_{i2t}B_{i2t}+ \\ &\sum_{i=1}^{N}\sum_{t=1}^{T}b_{k1t}B_{k1t}+\sum_{t=1}^{T}(hr_tH_t+l_tL_t)+\sum_{t=1}^{T}w_tW_t+\sum_{t=1}^{T}(hr'_tH'_t+l'_tL'_t)+\sum_{t=1}^{T}w'_tW'_t+ \\ &\sum_{l=1}^{L}\sum_{t=1}^{T}\sum_{k=1}^{K}C1_{l1t}Y_{k1t}+\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{i=1}^{I}C3_{j2t}Y_{i2t}+\sum_{i=1}^{N}\sum_{t=1}^{T}C5_{i2t}XD_{it}+\sum_{i=1}^{N}\sum_{t=1}^{T}C6_{i2t}XR_{it}+ \\ &\sum_{i=1}^{N}\sum_{t=1}^{T}hX_{i2t}XRI_{i2t} \end{aligned} \quad (1)$$

$$P_{i2t}+O_{i2t}+C_{i2t}+XR_{i2t}+B_{i2t}-B_{i2,t-1}+I_{i2,t-1}-I_{i2t}=D_{i2t}; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad (2)$$

$$P_{k1t}+O_{k1t}+C_{k1t}+B_{k1t}-B_{k1,t-1}+I_{k1,t-1}-I_{k1t}=\sum_{i=1}^{N}f_{ik}(P_{i2,t+LT}+O_{i2,t+LT}); \quad k=1,2,\ldots,K \quad t=1,2,\ldots,T \quad (3)$$

$$C_{k10}+I_{k10}=\sum_{i=1}^{N}f_{ik}(P_{i2,LT}+O_{i2,LT}); \quad k=1,\ldots,K \quad (4)$$

$$\sum_{i=1}^{N}(a_{i2j}P_{i2t}+U_{i2j}Y_{i2t})+m\,R_{jt}Y_{i2t}\le R_{jt}; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad j=1,2,\ldots,J \quad (5)$$

$$\sum_{i=1}^{N}(a_{i2j}O_{i2t})+m\,\beta_{jt}\,R_{jt}Y_{i2t}\le\beta_{jt}R_{jt}; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad j=1,2,\ldots,J \quad (6)$$

$$\sum_{i=1}^{N}(a_{k1l}P_{k1t}+U_{k1l}Y_{k1t})+m\,R'_{lt}Y_{k1t}\le R'_{lt}; \quad k=1,2,\ldots,K \quad t=1,2,\ldots,T \quad l=1,2,\ldots,L \quad (7)$$

$$\sum_{i=1}^{N}(a_{k1j}O_{k1t})+m\,\beta_{lt}\,R'_{lt}Y_{k1t}\le\beta_{lt}R'_{lt}; \quad k=1,2,\ldots,K \quad t=1,2,\ldots,T \quad l=1,2,\ldots,L \quad (8)$$

$$P_{k1t}+O_{k1t}\le MY_{k1t}; \quad k=1,2,\ldots,K \quad t=1,2,\ldots,T \quad (9)$$

$$P_{i2t}+O_{i2t}\le MY_{i2t}; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad (10)$$

$$W_t=W_{t-1}+H_t-L_t; \quad t=1,2,\ldots,T \quad (11)$$

$$W'_t=W'_{t-1}+H'_t-L'_t; \quad t=1,2,\ldots,T \quad (12)$$

$$\sum_{k=1}^{K}e_{k1}P_{k1t}\le fw'_t; \quad t=1,2,\ldots,T \quad (13)$$

$$\sum_{k=1}^{K}e_{k1}O_{k1t}\le\alpha'_t\,fw'_t; \quad t=1,2,\ldots,T \quad (14)$$

$$\sum_{i=1}^{N}e_{i2}P_{i2t}\le fw_t; \quad t=1,2,\ldots,T \quad (15)$$

$$\sum_{i=1}^{N}e_{i2}O_{i2t}\le\alpha_t fw_t; \quad t=1,2,\ldots,T \quad (16)$$

$$w_t\le w_{\max t}; \quad t=1,2,\ldots,T \quad (17)$$

$$w'_t\le w'_{\max t}; \quad t=1,2,\ldots,T \quad (18)$$

$$C_{i2t}\le C_{\max i2t}; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad (19)$$

$$B_{i2t}.I_{i2t}=0; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad (20)$$

$$B_{k1t}.I_{k1t}=0; \quad k=1,2,\ldots,K \quad t=1,2,\ldots,T \quad (21)$$

$$XR_{i2t}=XRI_{i2,t-1}-XD_{i2t}-XR_{i2t}+TR_{i2t}; i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad (22)$$

$$XD_{i2t}\le XD_{\max i2t}; \quad i=1,2,\ldots,N \quad t=1,2,\ldots,T \quad (23)$$

$XR_{i2t} \leq XR_{\max i2t}; \quad i = 1,2,...,N \quad t = 1,2,...,T$ (24)

$Y_{i2t} = \{0,1\}; \quad i = 1,2,...,N \quad t = 1,2,...,T$ (25)

$Y_{k2t} = \{0,1\}; \quad k = 1,2,...,K \quad t = 1,2,...,T$ (26)

2.2. The APP model and PM

In this section, we present an aggregate production planning model with preventive maintenance. This model is relevant to multi-period, multi-product, multi-machine, two-phase production systems.

2.2.1. Assumptions

- The quantity shortage at the beginning of the planning horizon is zero.
- The quantity shortage at the end of the planning horizon is zero.
- Maintenance decision variable, if maintenance to be performed, the decision variable is equal to one, and otherwise it is zero.
- There is a setup cost of producing a product only once at the beginning of a period, And the setup cost after a failure is not considered.
- If maintenance is not performed in period t, the time and cost of maintenance will not apply to the model, the failure costs will be considered in period t+1 instead, and downtime will be deducted from available machine capacity.

2.2.2. Model variables

In the second model, we have first model variables and appendix variable:

PMF_{lt}: The preventive maintenance decision variable of first-phase machine *l* in period *t*, a binary integer variable.

PMS_{jt}: The preventive maintenance decision variable of second-phase machine *j* in period *t*, a binary integer variable.

2.2.3. Parameters

In the second model, we have first model parameters and appendix parameters:

MTS_{jt}: The preventive maintenance time of second-phase machine *j* in period *t* (minutes).

MTF_{lt}: The preventive maintenance time of first-phase machine *j* in period *t* (minutes).

$C2_{l1t}$: Maintenance cost of first-phase machine *l* in period *t* ($).

$C4_{j2t}$: Maintenance cost of second-phase machine *j* in period *t* ($).

2.2.4. Second proposed Model

In the second model, we have first model Constraints and appendix Constraints:

The thirteenth term in (27) is failure cost for the first-phase. The fourteenth term in (27) is maintenance cost for the first-phase. The fifteenth term in (27) is failure cost for the second-phase. The sixteenth term in (27) is maintenance cost for the second-phase. The seventeenth term in (27) is disposed cost for the second-phase products. The eighteenth term in (27) is remanufactured cost for the second-phase products. The nineteenth term in (27) is inventory cost for the second-phase products.

Constraints (31) and (32) limit the regular time production to the available second group machines capacity, the overtime production to the available overtime and the preventive maintenance time for this group of machines respectively. Constraints (33) and (34) limit the regular time production to the available first group machines capacity, the overtime production to the available overtime and the preventive maintenance time for this group of machines respectively. Constraints (53) and (56) are the preventive maintenance decision variable for the both phase.

$$MinZ = \sum_{i=1}^{N}\sum_{t=1}^{T}(p_{i2t}P_{i2t} + o_{i2t}O_{i2t} + c_{i2t}C_{i2t}) + \sum_{k=1}^{K}\sum_{t=1}^{T}(p_{k1t}P_{k1t} + o_{k1t}O_{k1t} + c_{k1t}C_{k1t}) +$$
$$\sum_{t=1}^{T}\sum_{i=1}^{N}h_{i2t}I_{i2t} + \sum_{t=1}^{T}\sum_{k=1}^{K}h_{k1t}I_{k1t} + \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{J}r_{i2jt}Y_{i2t} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{l=1}^{L}r_{k1lt}Y_{k1t} + \sum_{i=1}^{N}\sum_{t=1}^{T}b_{i2t}B_{i2t} +$$
$$\sum_{i=1}^{N}\sum_{t=1}^{T}b_{k1t}B_{k1t} + \sum_{t=1}^{T}(hr_t H_t + l_t L_t) + \sum_{t=1}^{T}w_t W_t + \sum_{t=1}^{T}(hr'_t H'_t + l'_t L'_t) + \sum_{t=1}^{T}w'_t W'_t +$$
$$\sum_{l=1}^{L}\sum_{t=1}^{T}C1_{l1t}(1 - PMF_{l,t-1}) + \sum_{l=1}^{L}\sum_{t=0}^{T-1}C2_{l1t}PMF_{lt} + \sum_{j=1}^{J}\sum_{t=1}^{T}C3_{j2t}(1 - PMS_{j,t-1}) +$$
$$\sum_{l=1}^{J}\sum_{t=0}^{T-1}C4_{j2t}PMS_{jt} + \sum_{i=1}^{N}\sum_{t=1}^{T}C5_{i2t}XD_{i2t} + \sum_{i=1}^{N}\sum_{t=1}^{T}C6_{i2t}XR_{i2t} + \sum_{i=1}^{N}\sum_{t=1}^{T}hX_{i2t}XRI_{i2t} \quad (27)$$

$$P_{i2t} + O_{i2t} + C_{i2t} + XR_{i2t} + B_{i2t} - B_{i2,t-1} + I_{i2,t-1} - I_{i2t} = D_{i2t}; \quad (28)$$
$$i = 1,2,...,N \qquad t = 1,2,...,T$$

$$P_{k1t} + O_{k1t} + C_{k1t} + B_{k1t} - B_{k1,t-1} + I_{k1,t-1} - I_{k1t} = \sum_{i=1}^{N}f_{ik}(P_{i2,t+LT} + O_{i2,t+LT}); \quad (29)$$
$$k = 1,2,...,K \qquad t = 1,2,...,T$$

$$C_{k10} + I_{k10} = \sum_{i=1}^{N}f_{ik}(P_{i2,LT} + O_{i2,LT}); \qquad k = 1,...,K \quad (30)$$

$$\sum_{i=1}^{N}(a_{i2j}P_{i2t} + U_{i2j}Y_{i2t}) + PMS_{jt}MTS_{jt} + (1 - PMS_{j,t-1})m\,R_{jt} \le R_{jt}; \quad (31)$$
$$t = 1,2,...,T \qquad j = 1,2,...,J$$

$$\sum_{i=1}^{N}(a_{i2j}O_{i2t}) + (1 - PMS_{j,t-1})m\,\beta_{jt}\,R_{jt} \le \beta_{jt}R_{jt}; \quad t = 1,2,...,T \qquad j = 1,2,...,J \quad (32)$$

$$\sum_{i=1}^{N}(a_{k1l}P_{k1t} + U_{k1l}Y_{k1t}) + PMF_{lt}MTF_{lt} + (1 - PMF_{l,t-1})m\,R'_{lt} \le R'_{lt}; \quad (33)$$
$$t = 1,...,T \qquad l = 1,...,L$$

$$\sum_{i=1}^{N}(a_{k1j}O_{k1t}) + (1 - PMF_{l,t-1})m\,\beta'_{lt}\,R'_{lt} \le \beta'_{lt}R'_{lt}; \quad t = 1,2,...,T \qquad l = 1,2,...,L \quad (34)$$

$$P_{k1t} + O_{k1t} \le MY_{k1t}; \qquad k = 1,2,...,K \quad t = 1,2,...,T \quad (35)$$

$$P_{i2t} + O_{i2t} \le MY_{i2t}; \qquad i = 1,2,...,N \quad t = 1,2,...,T \quad (36)$$

$$W_t = W_{t-1} + H_t - L_t; \qquad t = 1,2,...,T \quad (37)$$

$$W'_t = W'_{t-1} + H'_t - L'_t; \qquad t = 1,2,...,T \quad (38)$$

$$\sum_{k=1}^{K}e_{k1}P_{k1t} \le fw'_t; \qquad t = 1,2,...,T \quad (39)$$

$$\sum_{k=1}^{K}e_{k1}O_{k1t} \le \alpha'_t\,fw'_t; \qquad t = 1,2,...,T \quad (40)$$

$$\sum_{i=1}^{N}e_{i2}P_{i2t} \le fw_t; \qquad t = 1,2,...,T \quad (41)$$

$$\sum_{i=1}^{N}e_{i2}O_{i2t} \le \alpha_t fw_t; \qquad t = 1,2,...,T \quad (42)$$

$$w_t \le w_{\max t}; \qquad t = 1,2,...,T \quad (43)$$

$$w'_t \le w'_{\max t}; \qquad t = 1,2,...,T \quad (44)$$

$$C_{i2t} \le C_{\max i2t}; \qquad i = 1,2,...,N \qquad t = 1,2,...,T \quad (45)$$

$$B_{i\,2t}.I_{i\,2t} = 0; \quad i = 1,2,\ldots,N \quad t = 1,2,\ldots,T \tag{46}$$

$$B_{k\,1t}.I_{k\,1t} = 0; \quad k = 1,2,\ldots,K \quad t = 1,2,\ldots,T \tag{47}$$

$$XR_{i\,2t} = XRI_{i\,2,t-1} - XD_{i\,2t} - XR_{i\,2t} + TR_{i\,2t}; \quad i = 1,2,\ldots,N \quad t = 1,2,\ldots,T \tag{48}$$

$$XD_{i\,2t} \le XD_{\max i\,2t}; \quad i = 1,2,\ldots,N \quad t = 1,2,\ldots,T \tag{49}$$

$$XR_{i\,2t} \le XR_{\max i\,2t}; \quad i = 1,2,\ldots,N \quad t = 1,2,\ldots,T \tag{50}$$

$$Y_{i\,2t} = \{0,1\}; \quad i = 1,2,\ldots,N \quad t = 1,2,\ldots,T \tag{51}$$

$$Y_{k\,2t} = \{0,1\}; \quad k = 1,2,\ldots,K \quad t = 1,2,\ldots,T \tag{52}$$

$$PMF_{lt} = \{0,1\}; \quad l = 1,2,\ldots,L \quad t = 1,2,\ldots,T \tag{53}$$

$$PMS_{jt} = \{0,1\}; \quad j = 1,2,\ldots,J \quad t = 1,2,\ldots,T \tag{54}$$

$$PMF_{l\,0} = 1; \quad l = 1,2,\ldots,L \tag{55}$$

$$PMS_{j\,0} = 1; \quad j = 1,2,\ldots,J \tag{56}$$

3. Harmony search

Harmony search (HS) algorithm was developed in an analogy with music improvisation process where music players improvise the pitches of their instruments to obtain better harmony (Lee & Geem, 2005). The steps in the procedure of HS are as follows (Mahdavi, 2007):

1. Initialize the problem and algorithm parameters.
2. Initialize the harmony memory.
3. New harmony improvisation.
4. Update the harmony memory.
5. Check the stopping criterion.

The pseudo-code of the original harmony search algorithm for the problem is shown in Figure 1:

Harmony search
Objective function f(x_i), i=1 to N
Define HS parameters: HMS, HMCR, PAR, and BW
Generate initial harmonics (for i=1 to HMS)
Evaluate f (x_i)
While (until terminating condition)
Create a new harmony: x_i^{new}, i=1 to N
If(U(0,1)≥HMCR),
$x_i^{new}=x_j^{old}$, where x_j^{old} is a random from {1,…,HMS}
Else if (U (0, 1) ≤PAR),
$x_i^{new}=x_L(i)+ U(0, 1)\times[x_U(i) - x_L(i)]$
Else
$x_i^{new}=x_j^{old} + BW[(2\times U(0,1))-1]$, where x_j^{old} is a random from {1,…,HMS}
end if
Evaluation f (x_i^{new})
Accept the new harmonics (solutions) if better
End while
Fine the current best estimates

Figure 1. Pseudo-code of the original harmony search

The search process stops if the some specified number of generations without improvement of the best known solution is reached. In our experiments we accepted Stop= 100.

4. Results

In order to evaluate the performance of the meta-heuristic algorithms, 30 test problems with different sizes are randomly generated for each model. The proposed models are coded with LINGO 8 software and using LINGO solver for solving the instances. Furthermore, for the small and medium sized instances of two phases APP with breakdown and PM, LINGO optimization solver is used to figure out the optimal solution and compared with HS results.
The proposed algorithm was programmed in MATLAB R2011a and all tests are conducted on a not book at Intel Core 2 Duo Processor 2.00 GHz and 2 GB of RAM.

4.1 Parameter calibration

Appropriate design of parameters has significant impact on efficiency of meta-heuristics. In this paper the Taguchi method applied to calibrate the parameters of the proposed methods namely HS algorithm. The Taguchi method was developed by Taguchi (Taguchi, 2000). This method is based on maximizing performance measures called signal-to-noise ratios in order to find the optimized levels of the effective factors in the experiments. In this subsection, the parameters for experimental analysis are determined. Table 1 lists different levels of the factors for HS. In this paper according to the levels and the number of the factors, the Taguchi method L_{25} is used for the adjustment of the parameters.

Table 1. Factors and their levels

Factors	Algorithm	Notations	Levels	Values
Harmony memory size		HMS	5	5,10,15,20,25
Harmony memory considering rate	HS	HMCR	5	0.7,0.75,0.8,0.85,0.9
Pitch-adjusting rate		PAR	5	0.1,0.15,0.2,0,25,0.3
Bandwidth		BW	5	0.2,0.5,0.8,0.9,0.99

Figure 2 show signal-to-noise ratios. Best Level of the factor for each algorithm is shown in table 2.

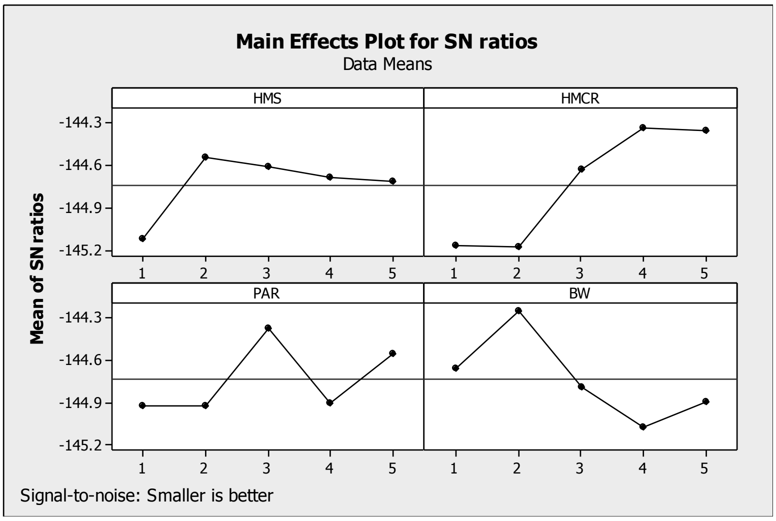

Figure 2. The signal-to-noise ratios for Harmony search

Table 2. Best level for parameters

Factors	Algorithm	Notations	Values
Harmony memory size		HMS	10
Harmony memory considering rate	HS	HMCR	0.85
Pitch-adjusting rate		PAR	0.2
Bandwidth		BW	0.5

4.2. Numerical results

Computational experiments are conducted to validate and verify the behavior and the performance of the harmony search algorithm to solve the APP model with breakdowns and PM. In order to evaluate the performance of the meta-heuristic algorithm, 30 test problems with different sizes are randomly generated for each model. These test problems are classified into three classes: small, medium and large size. Table 3 shows details of computational results obtained by solution method for all test problems for APP and breakdowns. Table 4 shows details of computational results obtained by solution method for all test problems for APP and PM.

Table 3. Details of computational results for APP and breakdowns

No	Prob. size	i.j.k.l.t	Lingo	Time(s)	HS	Time(s)
1	Small	2.1.2.1.3	7475340	1	7475340	6.3
2		2.1.2.2.3	7779851	1	7779851	14.3
3		2.2.2.1.3	8669761	1	8669761	7.7
4		2.1.3.1.3	7801244	2	7801244	60
5		2.1.4.1.3	8036987	2	8036987	1012.4
6		2.1.2.1.4	9874857	5	10091601	19.5
7		2.2.2.1.4	12103410	5	13602447	14.9
8		2.1.3.1.4	11338530	34	12898652.2	126.9
9		2.2.2.1.5	14701090	78	16131986.4	25.7
10		2.1.2.1.6	16912130	140	17804354.2	30.4
11	Medium	2.1.3.2.4	12240940	189	13643066	163.2
12		2.1.2.2.5	15181490	2320	17292317.4	26.4
13		4.1.2.1.3	---	---	16230083	31.2
14		3.1.2.1.5	---	---	22656268	60.8
15		2.1.4.1.5	---	---	18672008.2	3105.1
16		4.1.2.1.5	---	---	34718455.6	254.4
17		2.1.2.2.6	---	---	17012202	43.6
18		2.2.2.2.6	---	---	23034849.4	58
19		3.1.2.1.6	---	---	33017550.4	160.8
20		4.1.2.1.6	---	---	46314730.2	257.1
21		2.1.3.2.6	---	---	26547746.4	1848.7
22		2.1.2.1.8	---	---	34564240.8	50.8
23		2.1.2.2.8	---	---	31141170.2	109.4

No	Prob. size	i.j.k.l.t	Lingo	Time(s)	HS	Time(s)
24	Large	2.2.2.1.8	---	---	36790408.6	72.3
25		2.1.2.1.12	---	---	70206467.6	212.2
26		2.1.2.2.12	---	---	78730248.4	317.4
27		3.1.2.1.12	---	---	126332620.4	811.3
28		2.1.2.1.16	---	---	120606150.6	197.9
29		2.1.2.2.16	---	---	103364728.8	375.9
30		2.2.2.1.16	---	---	109493429	393.9

--- Means that a feasible and optimum solution has not been found after 3600 s of computing time.

Table 4. Details of computational results for APP and PM

No	Prob. size	i.j.k.l.t	Lingo	Time(s)	HS	Time(s)
1		2.1.2.1.3	6720124	1	6720124	10.6
2		2.1.2.2.3	7093831	1	7093831	18.2
3		2.1.3.2.3	7345570	1	7345570	32.6
4		2.1.4.1.3	7585061	1	7585061	824
5	Small	2.2.2.1.3	7594855	3	7594855	13.7
6		2.1.2.1.4	8522935	3	8522935	6.5
7		2.2.2.1.4	9939956	4	9939956	15.4
8		2.1.2.1.6	13931320	6	14142746	31.2
9		2.1.3.1.4	10185920	7	10525717.8	75.1
10		2.2.2.1.5	11858890	28	12088009.2	25.8
11		2.1.3.2.4	11042530	31	11420786	392.5
12		2.1.2.2.5	12824550	172	13627122.4	74
13		2.1.2.2.6	15105320	1035	16394869.4	37.7
14	Medium	2.2.2.2.6	16202530	2002	17340630.2	76.2
15		4.1.2.1.3	---	---	13750161	51
16		3.1.2.1.5	---	---	17321010	108.5
17		4.1.2.1.5	---	---	24351300.8	357.1
18		2.1.4.1.5	---	---	13170435.2	2213
19		3.1.2.1.6	---	---	27788070.6	105.9
20		4.1.2.1.6	---	---	36965194.4	493.5
21		2.1.3.2.6	---	---	19765159.2	1439.7
22		2.1.2.1.8	---	---	30847836.2	149.8
23		2.1.2.2.8	---	---	29067560.4	88.8
24	Large	2.2.2.1.8	---	---	31525137.6	114.5
25		2.1.2.1.12	---	---	63658096	118.3
26		2.1.2.2.12	---	---	63231299.6	312.5
27		3.1.2.1.12	---	---	91273845	676.1
28		2.1.2.1.16	---	---	83846588.6	120.2
29		2.1.2.2.16	---	---	76633081	214
30		2.2.2.1.16	---	---	85097596.6	226.1

--- Means that a feasible and optimum solution has not been found after 3600 s of computing time.

$$p_{k1t} \in [20,24], o_{k1t} \in [22,27], c_{k1t} \in [70,77], h_{k1t} \in [40,45], h_{k1t} \in [40,45], a_{k1l} = 1, u_{k1l} = 0.1,$$
$$r_{k1lt} \in [4,7], R'_{lt} \in [21000,40000], hr'_{t} \in [200,480], l'_{t} \in [200,480], w'_{t} \in [60,65], I_{k10} = 500,$$
$$w'_{0} = 3500, B_{k10} = 0, e_{kl} = 0.2, \alpha'_{t} = 0.2, \beta'_{lt} = 0.5, w'_{\max t} \in [3000,7000], D_{i2t} \in [6000,24000],$$
$$p_{i2t} \in [20,25], o_{i2t} \in [22,27], c_{i2t} \in [100,106], h_{i2t} \in [60,67], a_{i2j} \in [0.4,0.5], u_{i2j} = 0.2,$$
$$r_{i2jt} \in [10,15], R_{jt} \in [21000,40000], hr_{t} \in [200,460], l_{t} \in [200,460], w_{t} \in [61,64], I_{i20} = 500,$$
$$w_{0} = 3500, B_{i20} = 0, e_{i2} = 0.4, \alpha_{t} = 0.2, \beta_{jt} \in [0.4,0.5], f \in [120,190], w_{\max t} \in [3000,7000],$$
$$C_{\max it} \in [2000,9500], f_{ik} = 2, C1_{l1t} \in [100000,220000], C2_{l1t} \in [10000,50000],$$
$$C3_{j2t} \in [100000,220000], C4_{j2t} \in [10000,50000], C5_{i2t} \in [11,14], C6_{i2t} \in [4,7],$$
$$MTS_{jt} \in [1500,5000], MTF_{lt} \in [1500,5000], TR_{i2t} \in [300,800], XD_{\max i2t} \in [300,600],$$
$$XR_{\max i2t} \in [400,650], hX_{i2t} \in [60,65];, m = 0.1, LT = 1.$$

4.3. Preventive maintenance effect on the objective function

We used HS algorithm to shown Preventive maintenance effective on the objective function. Table 5 shows details of computational results between Aggregate production planning with breakdowns and Aggregate production planning with Preventive maintenance, also amount of cost reduction and percentage of cost reduction. Also Figure 3 depicts comparison between solution quality of the Aggregate production planning with breakdowns and Aggregate production planning with Preventive maintenance of the instances. So the objective values obtained by Aggregate production planning with Preventive maintenance are better from Aggregate production planning with breakdowns results.

Table 5. Details of computational results between APP with breakdowns and APP with PM

No	i.j.k.l.t	APP and breakdowns	APP and PM	Amount of cost reduction	Percentage of cost reduction
1	2.1.2.1.3	7475340	6720124	755216	10%
2	2.1.2.2.3	7779851	7093831	686020	9%
3	2.1.3.2.3	7801244	7345570	455674	6%
4	2.1.4.1.3	8036987	7585061	451926	6%
5	2.2.2.1.3	8669761	7594855	1074906	12%
6	2.1.2.1.4	10091601	8522935	1568666	16%
7	2.2.2.1.4	13602447	9939956	3662491	27%
8	2.1.2.1.6	17804354.2	14142746	3661608.2	21%
9	2.1.3.1.4	12898652.2	10525717.8	2372934.4	18%
10	2.2.2.1.5	16131986.4	12088009.2	4043977.2	25%
11	2.1.3.2.4	13643066	11420786	2222280	16%
12	2.1.2.2.5	17292317.4	13627122.4	3665195	21%
13	2.1.2.2.6	17012202	16394869.4	617332.6	4%
14	2.2.2.2.6	23034849.4	17340630.2	5694219.2	25%
15	4.1.2.1.3	16230083	13750161	2479922	15%
16	3.1.2.1.5	22656268	17321010	5335258	24%
17	4.1.2.1.5	34718455.6	24351300.8	10367155	30%
18	2.1.4.1.5	18672008.2	13170435.2	5501573	29%
19	3.1.2.1.6	33017550.4	27788070.6	5229479.8	16%
20	4.1.2.1.6	46314730.2	36965194.4	9349535.8	20%
21	2.1.3.2.6	26547746.4	19765159.2	6782587.2	26%
22	2.1.2.1.8	34564240.8	30847836.2	3716404.6	11%
23	2.1.2.2.8	31141170.2	29067560.4	2073609.8	7%
24	2.2.2.1.8	36790408.6	31525137.6	5265271	14%
25	2.1.2.1.12	70206467.6	63658096	6548371.6	9%
26	2.1.2.2.12	78730248.4	63231299.6	15498949	20%
27	3.1.2.1.12	126332620.4	91273845	35058775	28%
28	2.1.2.1.16	120606150.6	83846588.6	36759562	30%
29	2.1.2.2.16	103364728.8	76633081	26731648	26%
30	2.2.2.1.16	109493429	85097596.6	24395832	22%
			Average	**7734213**	**18%**

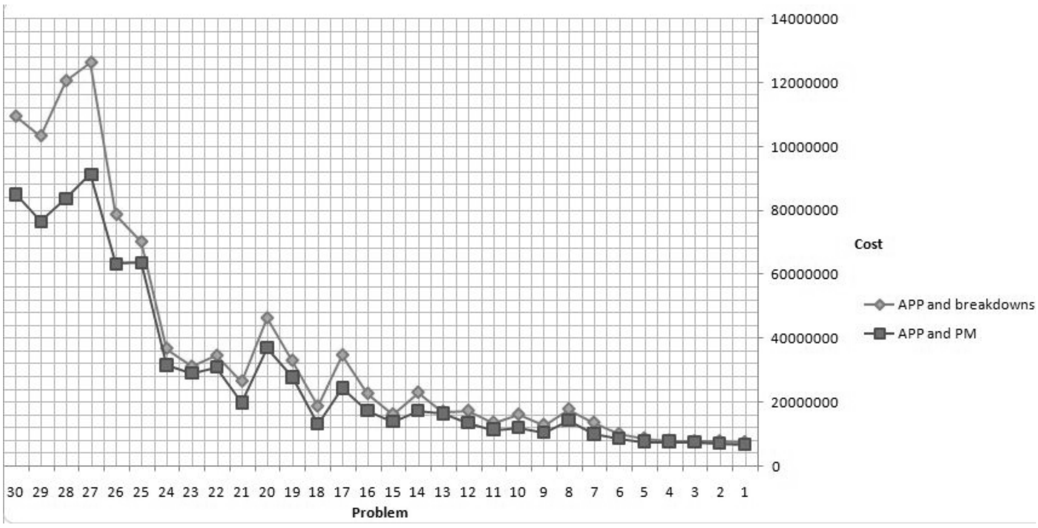

Figure 3. Comparison between solution quality of the APP with breakdowns and APP with PM

5. Conclusion

The purpose of this paper is to formulate and solve aggregate production planning model with return products, machine breakdowns and preventive maintenance for two phase production systems in which the objective function is to minimize the costs of production over the planning horizon. We develop a mixed integer linear programming model that can be used to compute optimal solution for the problems by an operation research solver. Due to the complexity of the problem harmony search algorithm was used to solve the problem. Moreover, an extensive parameter setting with performing the Taguchi method was conducted for selecting the optimal levels of the factors that effect on algorithm's performance. Also the computational results show that, the objective values obtained by APP with PM are better from APP with breakdowns results. We have 18% improvement in the second model than the first model. The research is continued in direction of further extension of the proposed APP and PM model by representing additional sources of uncertainty such as demand, return products, breakdowns, etc. Also, developing new meta-heuristic algorithms to make better solutions can be suggested.

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