THE ROLE OF RISK IN THE VALUATION OF TAX SHIELD

Lucia MICHALKOVA [a*], ***Tomas KLIESTIK*** [a],

[a] *University of Zilina, Slovakia*

ABSTRACT

Finding the optimal capital structure is one of the crucial aspects of corporate finance. The value of the company / project, which is funded by equity and debt, is higher because of the tax advantage of debt - interest tax shield. Debt and the interest tax shield are not risk-free as assumed by Modigliani and Miller (1963); they are subject to business risk - operational, credit and default risk. The aim of this paper is to analyse, compare and classify models of the present value of tax shield to the risk profile of debt and tax shield. In addition to the comparison, the Fernandez (2006) algorithm was used to identify the risk of this cash flow. The results indicate that five different discount rates according to risk profile can be used to quantify the present value of tax shield: risk-free interest rate, cost of debt, unlevered cost of capital, leveraged cost of equity and option-based stochastic discount factor.

KEYWORDS: *tax shield, debt, risk, default, valuation*

1. INTRODUCTION

The capital structure is one of the most discussed topics in corporate finance. This variable is subject to both microeconomic and macroeconomic factors (Hannapi, 2018). It is assumed that debt is cheaper than equity because borrowers are guaranteed debt repayment. On the other hand, equity holders do not have a guaranteed dividend payment. This imbalanced risk profile of debt and equity also causes imbalances between rates of return on both types of capital (Valaskova et al., 2015).

The second aspect of debt financing is the tax benefit of a debt called a tax shield. This is more advantageous for businesses to pay interest on pre-tax profits and reduce tax liability than pay dividends on net profit. This creates a net debt preference over equity, as suggested by the Trade-off theory developed by Modigliani and Miller (1963). However, leverage increases the risk of default and cost of capital. This aspect of debt risk is reflected in the golden rules of the capital structure used in Central European countries (the golden rule of financing or the golden rule of risk compensation) (Emerling and Wojcik-Jurkiewicz, 2018).

The tax shield has a significant impact not only on the optimization of the capital structure, but also on key indicators of financial performance: profit, profitability, cash flow and the market value of company or the market value of share. It is the market value of the enterprise and its maximization that is considered to be the main objective of any enterprise. Graham (2000) found that the tax shield represented on average 9.7% of the value of US businesses. Kemsley and Nissim (2002) achieved similar results by regression analysis; 10% of the value of company respectively 40% of the value of debt. Korteweg (2010) and Van Binsbergen et al. (2010) argue that the value of tax shield is lower and reaches only 5.5% of the value of company. Doidge and Dyck (2015) have researched Canadian firms and they found that the share of the tax shield in the corporate valueis lower than in US firms; only 4.6%. Ko and Yon (2011) found similar results on the Korean market as Korteweg (2010): 5.2% share in the value of the company. Clemente-Almendros and Sogorb-

[*]Corresponding author. E-mail address: *lucia.michalkova@fpedas.uniza.sk*

Mira (2017) focused on Spanish listed companies; their estimate is higher than in the above-mentioned research, 6.4% of the value of the company. These results are influenced not only by the leverage of the companies surveyed and the tax policies of the countries, but also by the riskiness of these markets.
As previous research shows, the (interest) tax shield has a substantial impact on the corporate value, thus on the value of shares (Olah et al., 2017). Quantification of the present value of tax shield with respect to specific aspects of the company plays a key role in the correct valuation of the share of the leverage company. Since the creation of the first method of calculating the tax shield by Modigliani and Miller (1963), more than two dozen models of the present value of tax shield have been created. These models involve risk in the value of the tax shield in various ways or abstract from certain risks. The aim of this paper is to analyze, compare and classify the models of present value of tax shield according to the risk profile of debt and tax shield. In addition to comparing these models, Fernandez algorithm (2006) was used to identify the risk of this cash flow.
The paper is organized as follows: in Literature review this is the definition of the tax shield and the models of the present value of tax shield are briefly summarized. The Fernandez algorithm (2006) is presented in the Methods chapter. Identification of risks associated with the tax shield and classification of the models of present value of tax shield and application of Fernandez algorithm is a part of chapter Results and Discussion. In conclusion, we point out some shortcomings of current models and state the risks involved in these models.

2. LITERATURE REVIEW

2.1 Definition of tax shield

Modigliani and Miller (1958) created the first universally theory of capital structure. The model is also called capital structure irrelevance theorem; the corporate value does not change depending on the form of financing (by debt or equity). In addition, the model does not assume the existence of taxes. The modified Modigliani and Miller model (1963), also the MM model, deleted the unrealistic assumption of the original model, company value increases due to the growth of corporate leverage. The value of tax shield (tax advantage, tax benefit) arises from the tax deductibility of interest. Each company could increase its corporate value due to the growth of debt; however, with leverage growth, the risk of default, represented by the cost of financial distress, also increases (trading-off theory of capital structure).
Investopedia (2018) defines a tax shield as: "*A tax shield is a reduction in taxable income for an individual or corporation achieved through claiming allowable deductions such as mortgage interest, medical expenses, charitable donations, amortization and depreciation. These deductions reduce a tax payer's taxable income for a given year or defer income taxes into future years. Tax shields lower the overall amount of taxes owed by an individual taxpayer or a business. Other authors are focused on the interest tax shield*." Brealey, Myers and Allen (2010) define the tax shield in a narrower sense: *"tax savings resulting from deductibility of interest payments*." Damodaran (2010) agreed with the previous: *"Interest is tax-deductible, and the resulting tax savings reduce the cost of borrowing to firms."*
Similar to Modigliani and Miller (1963), interest is a tax deductible expense affects the earnings before tax, hence the amount of corporate tax liability. It means that the corporate value may change due to the existence of an interest tax shield. The value of tax shield is given as a product:

$$\mathrm{TS} = \mathrm{T} \times \mathrm{k_D} \times \mathrm{D} \tag{1}$$

Where: TS - value of tax shield, T - corporate tax rate, k_D - cost of debt, D - market value of debt.
A necessary condition for non-zero value of tax benefit is the positive profit before tax that covers the interest paid.

In addition to the method mentioned in Eq. (1) there are also four different formulations of the value of tax shield. If it is assumed that the value of debt is constant and perpetual, the corporate tax shield depends only on the corporate tax rate and the value of debt. Hence, the present value of tax shield is equal to the discounted value of Eq. (1). As the discount factor, the cost of debt k_D is used, since the risk of the tax shield (hereinafter TS) should be the same as the risk of interest paid.

$$\mathrm{PV(TS)} = \frac{\mathrm{T} \times \mathrm{k_D} \times \mathrm{D}}{\mathrm{k_D}} = \mathrm{T} \times \mathrm{D} \tag{2}$$

Where: $PV(TS)$ is the present value of interest tax shield.
The previous Eq. (1) and (2) show that the tax advantage is affected by three factors: corporate tax rate; cost of debt and the value of debt (the market value of debt). Liu (2009), in contrast to the other formulae, argues that tax shield is a variable influenced by four factors: "*Tax shield is a function of four variables 'net income, interest rate, debt, and tax rate.' However, the value of the MM tax shields only includes two variables ''debt and tax rate," is independent of interest rate, and cannot be true.*"
Velez - Pareja (2013) is in favour of the definitions of other authors: "*Tax shields or tax savings TS, are a subsidy that the Government gives to those who incur in deductible expenses. All deductible expenses are a source of tax savings. This is, labour payments, depreciation, inflation adjustments to equity, rent and any expense if they are deductible.*"
If the main source of tax savings is interest paid, the company achieves the tax shield. If the operating profit (EBIT) plus other income are sufficient to offset the company interest paid, the value of tax benefit is equal to the product of tax rate and the financial expenses (FE). If the operating profit plus other income (OI) is less than the value of financial expenses, the corporate income tax is not paid by the company. Nevertheless the tax shield is earned and its value is equalled to corporate tax rate times EBIT plus OI according to Eq. (3).

$$\mathrm{TS} = \mathrm{T(EBIT + OI)} \text{ if } 0 < \mathrm{EBIT + OI} < \mathrm{FE} \tag{3}$$

If the sum of *EBIT* and *OI* is negative, tax savings do not arise because the company does not pay any tax. In sum, all possible cases are given in Eq. (4).

$$\mathrm{TS} = \begin{Bmatrix} \mathrm{T \times FE} \text{ if } \mathrm{EBIT + OI} \geq \mathrm{FE} \\ \mathrm{T \times (EBIT + OI)} \text{ if } 0 \leq \mathrm{EBIT + OI} \leq \mathrm{FE} \\ 0 \text{ if } \mathrm{EBIT + OI} < 0 \end{Bmatrix} \tag{4}$$

This is significance for further research; most of the literature dealing with the issue of tax shields is based on Eq. (2). It also means that both new businesses and start-ups can achieve partial tax savings, despite the fact that *EBIT* and *OI* cannot cover the value of financial expenses. Eq. (4) indicates that the value of tax shield is a function of *EBIT* plus *OI* and not a function of the net income as Liu (2009) argued in his theory.
Calculation of the value of tax shield is not only based on the interest, but in practice it is much more complicated. The deductible expenses for tax calculation also include loss carried forward, deferred tax liability, tax prepayments, etc. If the loss carried forward is allowed, according to Velez-Pareja (2016) the tax shield does not reduce the tax liability in just one period, but its part is transferred to another future period when the loss is carried forward. In simplified terms, the value of tax shield is equal to the formula

$$\mathrm{TS} = \mathrm{Maximum}(\mathrm{T} \times \mathrm{Minimum}\left(\mathrm{EBIT_{adj}}, \mathrm{FE} + \mathrm{LCF_t} - \mathrm{LCF_{t-1}}\right), 0) \tag{5}$$

where LCF_{t-1} or LCF_t is loss carried forward at period *t (t-1)* and $EBIT_{adj}$ is *EBIT* plus other income (*OI).*
Miller (1977) explored the impact of personal taxes on capital structure, he concludes that there is no benefit from debt financing because the effect of corporate taxes and personal taxes is mutually interrupted. Therefore, according to the author, the value of the tax shield is zero. Velez-Pareja (2017) states that Miller (1977) approach might be incorrect and has several inconsistencies. First of all, the taxation of interest received by debt holders is irrelevant to the value of the tax shield because the valuation method does not take into account the personal taxes of the debt-holders. Unlike Miller (1977), it is assumed that the debt holders and the holders of equity are different people. The net tax shield is therefore the difference between the tax shield and the personal tax on dividends, as shown in the equation (6).

$$\text{Net TS} = \mathrm{k_D B}\left[\mathrm{T_{ps}} + \mathrm{T_c}\left(1 - \mathrm{T_{ps}}\right)\right] - \mathrm{X_t}(1 - \mathrm{T_c})\mathrm{T_{ps}} \tag{6}$$

Where B is book value of debt, T_{ps} is personal tax rate on dividends, T_c is corporate tax rate and X_t is earnings before interest and taxes or operating income.
These definitions of the tax shield are contradictory and give different perspectives on its value.

2.2 Risk and present value of the tax shield
In this chapter we focus on the analysis and classification of theories to quantify the present value of tax shield. The criterion for the breakdown of these theories is the nature of a debt policy, which is part of financial management of a company. The debt policy is a source of differences between the various theories.
Modigliani and Miller (1963) model is the first of the analyzed theories. According to the model assumptions, the company can lend and borrow capital on a perfect capital market at risk free interest rate and it assumed constant market value of debt. Tax savings (tax shield) is risk free and the appropriate discount rate is risk free interest rate r_f. The present value calculation is similar to that in Eq. (2).

$$\mathrm{PV(TS)} = \frac{\mathrm{T \times r_f \times D}}{\mathrm{r_f}} = \mathrm{T \times D} \tag{7}$$

The previous model is based on the assumptions of the efficient (perfect) capital market, so there are some limitations and more appropriate theories assuming the riskness of debt. Similarly, to Modigliani and Miller (1963), the other models mentioned above assume that the value of debt is fixed and does not change over a given period. The risk of debt determines the appropriate discount rate and its choice varies according to the authors' opinion. Myers (1977) and similarly Ruback (2002) assume that both the tax advantage and the debt are equally risky, therefore they are discounted by the cost of debt k_D similarly to the Eq.(2). Kaplan and Ruback (1995), on the other hand, assume that both the interest tax shield and the company's cash flow share the same systematic risk, these components of the company value (project value) are discounted at the cost of unlevered company (k_U).

$$\mathrm{PV(TS)} = \frac{\mathrm{T \times k_D \times D}}{\mathrm{k_U}} \tag{8}$$

The assumption of a fixed amount of debt is rather simplifying, as the company does not know with certainty the future level of debt. Real debt policy should be based on the fixed debt ratio (*D / V*) instead of constant value of debt, where *V* is the sum of the market value of the debt and equity of a company. If *D = const*., the future interest tax shields are deterministic because their values are known at time *t = 0*. The present value of tax benefits may vary in accordance to a change in the discount rate reflecting both microeconomic and macroeconomic conditions. If the constant leverage ratio is assumed, future tax shields have stochastic character and their values may be only estimated. Quantification of the present value of tax shield, assuming constant leverage is ambiguous because the individual authors differently assess the risk of a tax advantage.

The first model based on a constant leverage ratio is a model of Miles and Ezzell (1985), also called the ME model, in which it assumed that the value of debt is known in the first period; it is deterministic. This cash flow is unknown in other periods, because the value of debt is stochastic. The present value of tax shield is deterministic in the first period; in other periods is stochastic. The appropriate discount rate for the first period is the cost of debt; in subsequent years, it is unlevered cost of equity.

$$PV(TS) = \frac{1 + k_U}{1 + k_D} \frac{T \times k_D \times D_{t-1}}{k_D} \qquad (9)$$

The Harris and Pringle (1985) model is based on the ME model; the company follows a financial strategy that maintains the constant leverage ratio. Capital structure is continually changed to achieve the constant leverage ratio. The debt is stochastic because its value varies throughout the forecast period and is unknown in all future periods, including the first one; the tax shields are also stochastic. The unlevered cost of capital k_U is appropriate discount rate which reflects the riskiness of the cash flows.

$$PV(TS) = \frac{T \times k_D \times D}{k_U} \qquad (10)$$

Arzac and Glosten (2005), based on the ME approach, eliminate the discount rate by using a stochastic discount rate („*pricing kernel*").On the other hand, Liu (2009) suggests to divide the value of tax benefit into two parts: *unearned tax shield* and *earned tax shield* depending on whether the interest rate is lower or higher than return on investment (ROI). The author himself noted that the theory is inconsistent with other theories.

The Fernandez (2004a) model for quantification of the value of tax shield is different than others mentioned above. He argued that this model is independent of the nature of debt policy (Fernandez, 2004a). The value of tax shield is not equal to the present value of tax advantages, but the value of tax shields (*VTS*) is quantified as the difference between the present values of two cash flows of each with different risk: the present value of taxes paid by unlevered company and the present value of taxes paid by levered company.

Cooper and Nyborg (2006) argued that the Fernandez (2004a) model is based on the combination of two different approaches (MM and ME) and therefore the value of tax shield is equal to the present value of tax shield. Based on Fernandez approach, the authors found out that the value of tax shield is identical to the Harris and Pringle (1985) model in the case of perpetuity. Cooper and Nyborg (2006) also set out four different debt policy alternatives in terms of risks:

1. Operating cash flow is a risky perpetuity combined with a constant amount of debt.
2. Constant proportional market-value leverage.
3. Any arbitrary non-constant leverage policy, with tax benefits from debt that have the same risk as the operating FCF (free cash flows).
4. Any arbitrary non-constant leverage policy, with tax benefits that are as risky as the debt.

The first approach is the classical approach of Modigliani and Miller (1963). On the contrary, the second approach represents the model of Miles and Ezzell (1980). As reported in Cooper and Nyborg (2006), tax benefits have the same risk as operating cash flow (often referred to as *EBIT*). Arbitrary debt policy assumes that tax savings have the same risk as operating cash flow because there is uncertainty that an enterprise will not be able to pay tax in the future. This approach is referred to as extended Miles and Ezzell (extended ME). Unlike the classic ME model, the expanded ME has no assumption of a stable leverage ratio; leverage can vary over time. The last debt policy mentioned above is referred to as the extended Modigliani and Miller (extended MM). It has similar assumptions to the classical model of Modigliani and Miller (1963); in addition to the riskless of the tax shield (constant debt value), it also assumes an increase in debt in line with the growth in operating cash flow. This is very unlikely and unrealistic.

Fernandez (2004b) and (2005) modified his model. The value of tax shield quantified by his modified model should depend only on the nature of the stochastic process of the net increase of debt. It should not depend on the nature of the stochastic process of the free cash flow.

On the other hand, there are tax shield models based on the book value of the debt. Unlike previous models, they can be applied to the valuation of non-listed companies from emerging markets. Fernandez (2007) assumed that the debt policy is set according to the book leverage ratio. Marciniak (2003) suggested *decomposition method* for business valuation. In comparison with other models, this method uses the book value of debt and all cash flows are discounted at the same discount rate (cost of the levered equity rather than WACC), which prevents the formation of circularity problem. In addition to the models mentioned Velez-Pareja (2013) also examined the value of debt tax shield and assumed the book value of debt. He found that the tax interest shield is a function of operating profit (EBIT).

Velez - Pareja (2010) states that the riskiness of the tax shield is routinely examined from the perspective of the riskiness of debt. Based on formula in Eq. (4), however, he found that the tax benefit is associated with operational risk. The author states that there are three sources of risk associated with the tax shield: the default risk in debt, the risk of market debt costs and operational risk. The default risk in debt arises when an enterprise defaults in taxes, i.e. the enterprise does not pay taxes. Conversely, if an enterprise does not have enough cash to pay interest but pays taxes, then only the debt is risky, but the tax shield is not. The risk of market debt costs is associated with the variable k_D. The market cost of debt may change in relation to systematic risk. If the cost of the debt were contractual and unchanged, the risk would not arise. However, the contraction rate is mostly linked to the market rate of cost of debt. First of all, the contractual rate can only be fixed for a certain period of time, and then it changes in line with market conditions. Contractual rate is not linked to market conditions in specific cases only.

In accordance with the given sources of risk, none of the previous discount rates is an appropriate rate to quantify the present value of tax shield. Velez-Pareja (2010) proposes a rate combining debt and operational risk.

Models based on option value quantification are currently the most up-to-date models of tax shield value. Such models include, for example, models by Couch et al. (2012), Molnar and Nyborg (2011) or Grinblatt and Liu (2008). The disadvantage of these models is their computational difficulty; for this reason, these models are relatively less well known and less used in practice.

Couch et al (2012) combined the traditional model of Modigliani and Miller (1963) with the evaluation of the barrier option. They developed three strategies for valuing the tax shield assuming default risk: constant debt, delayed debt and debt refinancing. As stated by the authors, the risk profile of the tax shield is different from the risk profile of the debt, because by default the debt value is positive, but the tax shield usually terminates. Equally, equity risk is different from tax shield risk, because equity cash flow is more variable than tax shield.

In the first strategy, the company constantly maintains the same value of debt, there is default is conditioned by the income (operating income) of the enterprise. The present value of tax shield is given as the price of the cash-at-hit-or-nothing barrier option. The option makes a payout only when the underlying asset reaches a barrier; determined by the value of interest coverage. The second strategy is that the company has no debt, but it plans to borrow if income reaches the set value. This strategy appropriately describes start-ups whose tax shield will be created in the future. The latest strategy is related to debt refinancing and change of leverage, if required. This strategy is a combination of two barrier options; one describes default and the other describes debt refinancing.

3. METHODS

This chapter is aimed at describing methods used to investigate the value of tax shields according to debt policy and risk profile. Formulae of selected models mentioned in the previous chapter are shown in Table 1.

Table 1 Formulae of tax shield valuation models

Author	Formula
Modigliani and Miller (1963)	$PV(TS) = TD$
Myers (1977)	$PV\ (TS) = \frac{Tk_D D_{i-1}}{k_D}$
Miller (1977)	$PV(TS) = 0$
Miles and Ezzell (1985)	$PV(TS) = \frac{1 + k_U}{1 + k_D} \frac{Tk_D D_{t-1}}{k_U}$
Harris a Pringle (1985)	$PV\ (TS) = \frac{Tk_D D_{i-1}}{k_U}$
Marciniak (2003)	$PV(TS) = \frac{k_D BT}{k_E}$
Fernandez (2005)	$VTS = T \times D_0 + T \times PV[\Delta\ D]$
Cooper and Nyborg (2006)	$VTS = \frac{Tk_D D}{k_U - g}$
	$VTS = \frac{Tk_D D}{k_D - g}$
Damodaran (2006)	$PV(TS) = \frac{(k_U DT - (k_D - r_f) D(1 - T))}{k_U}$
Practioner´s method*	$PV(TS) = \frac{(k_D DT - (k_D - r_f) D)}{k_U}$
Fernandez (2007)*	$VTS^{BV} = \frac{Tk_U B}{k_U - g}$
Liu (2008)	$\text{Tax shield capacity} = \frac{r_f}{ROI} TD + \left(1 - \frac{r_f}{ROI}\right) TD$
Couch et al. (2012)	$VTS\ [\text{the first strategy}] = \frac{T \times C}{r_f}\left[1 - \left(\frac{H}{X}\right)^{b_p}\right]$
	$VTS\ [\text{the second strategy}] = \frac{T \times C^*}{r_f}\left[1 - \left(\frac{H^*}{K}\right)^{b_p}\right] \times \left(\frac{K}{X}\right)^{b_m}$
	$VTS\ [\text{the third strategy}] = \frac{T \times C}{r_f}(1 - P_{HK} - Q_{HK}) + \frac{T \times C^*}{r_f}\left(1 - \left(\frac{H^*}{K}\right)^{b_p}\right) Q_{HK}$
Velez-Pareja (2013)	$TS = Max(T \times Min(EBIT + OI, FE), 0)$
Velez-Pareja (2016)	$TS = Maximum(T.Minimum(EBIT_{adj}, FE + LCF_t - LCF_{t-1}), 0)$
Velez-Pareja (2017)	$\text{Net TS} = k_D B[T_C(1 - T_{pS})] - X(1 - T_C) T_{pS}$

*derived in Fernandez (2007)

Where: k_E – levered cost of equity, $\overline{ROI}$ – avereage annual return on investments (assets), g – growth rate, C – interest expense, $\left(\frac{H}{X}\right)^{b_p}$ - down-and-in, cash-at-hit-or-nothing, single-barrier option valuation formula, X – current value of *EBIT*, C^*- the future value of interest expense , H^* - the value of barrier according to C^*, $\left(\frac{H^*}{K}\right)^{b_p}$ - down-and-in, cash-at-hit-or-nothing, single-barrier option valuation formula for the second strategy, $\left(\frac{K}{X}\right)^{b_m}$- up-and-in, cash-at-hit-or-nothing, single barrier option valuation formula,
Cost of capital was quantified according to the Damodaran methodology (Weissova et al., 2015). In the non-traded business environment, information on market values of equity and debt is not available. This criterion would eliminate almost all models of the value of the tax shield except a small set of models based on the book value of debt. In the case of this analysis, however, it is assumed that market and book values are equal; Fernandez (2006) also mentions this possibility. Return on investment (ROI) is quantified as the average annual return on assets. Growth of the enterprise is calculated similarly: as the average annual growth rate of *EBIT* over the reference period. In Table 1, the discount factor is not in the Velez-Pareja (2013) model and in the Velez-Pareja (2017). Velez-Pareja (2010) suggested the discount rate includes both market risk and default risk as follows

$$\psi = W_{dr}K_D + W_{EBIT+OI}K_U \tag{11}$$

The weight W_{dr} of represents the share of enterprises that have gone bankrupt (dying) to the total number of enterprises in Slovakia in 2016; on the contrary, the weight $W_{EBIT+OI}$ is the share of enterprises that did not bankrupt to the total number of enterprises in the given period. Data were collected from the Eurostat database (2018). For the Velez-Pareja personal taxation model (2017), it presents two different appropriate discount rates: cost of debt or unlevered cost of equity. Variables of Couch et al. model (2012) is quantified according to the author's formulae in their paper.
Fernandez (2006) worked on the existence of so-called "cost of leverage" in analysing models of value of tax shield. This cost is an approximation of decision making between the debt, risk-free debt or equity financing. The author expressed the hypothesis that if this cost does not exist, the value of tax shield (the present value of tax shield) is equal to the maximum value of interest tax shield, i.e. value according to Modigliani & Miller (1963). Conversely, if this cost exists, which is real in the imperfect markets, the value of tax shield is lower than the value of debt times corporate tax rate. Fernandez also sets the second condition for analysis, namely the theoretical absence of taxes. This means that in a hypothetical country, businesses may not be taxed at corporate level, but risk and cost of leverage may still be present. Therefore, the value of tax shield for zero taxation and cost of leverage is less than zero; if there is no cost of leverage, the value of tax shield is zero. Clearly, these conditions are written in Table 2.

Table 2 Necessary conditions of the analysis

Necessary conditions	Leverage costs ≠ 0	Leverage costs = 0
Corporate tax > 0	< DT	=DT
Corporate tax = 0	< 0	= 0

Source: adapted from Fernandez (2007)

5. RESULTS AND DISCUSSION

Based on the Literature review chapter, we can distinguish several different discount rates that reflect the riskiness of the tax shield:

- risk-free interest rate - Modigliani and Miller model (1963) for perfect capital markets,
- cost of debt - the tax shield is as risky as debt, the constant value of debt,
- levered cost of equity - the tax shield takes into account equity risk with respect to leverage (Marciniak, 2003),
- unlevered cost of equity - tax shields are uncertain and taking into account the systemic risk of unlevered equity,
- stochastic discount factor - the value of the tax shield can be certain or uncertain, the risk varies between debt risk and unlevered equity risk,
- discount factor for the debt risk and operational risk - Velez-Pareja (2016) model.
- valuation based on options - takes into account default and removes the decision on the appropriate discount factor.

The present value of tax shield and the value of the tax shield are subject to the impact of three more of the risks - operational risk (venture will not have enough income to pay the interest paid), the risk of debt (enterprise will not be able to repay principal repayments), the risk of market cost of debt (cost of capital change), corporate default risk (risk of bankruptcy) and the risk of default in taxes (company does not have enough cash to pay taxes). These risks are reflected in the formulas for calculating the present value of tax shield. The impact of various risks on the value of the tax shield can be compared by the Fernandez (2006) algorithm.
The first step of this analysis was the calculation of input variables. The financial information from the financial statements of one Slovak enterprise from 2010-2016 was used for their calculation. According to Velez-Pareja (2010), the information needed for the calculation of the discount rates was drawn from Eurostat. These input data are shown in Table 3.

Table 3 Input data for analysis

Variable	Input data	Variable	Input data
Interest-bearing debt in time t-1	62 502 (Eur)	Average company growth	1.190%
Interest-bearing debt in time t	26 263 (Eur)	$\overline{\mathrm{ROI}}$	1.061%
Corporate tax rate	21%	EBIT + OI	14 939 (Eur)
Tax rate for dividends	7%		
Riskless interest rate	0.544%	Interest paid	1 950 (Eur)
Cost of debt	7.425%	W_{dr}	2.446%
Unlevered cost of equity	13.941%	$W_{EBIT+OI}$	97.554%
Levered cost of equity	18.017%	H^*	2076.80 (Eur)
C^*	2380.58 (Eur)	X	14 939 (Eur)
$\left(\frac{H}{X}\right)^{b_p}$	0.981666	$\left(\frac{H^*}{K}\right)^{b_p}$	0.980292
$\left(\frac{K}{X}\right)^{b_m}$	0.714373	P_{HK}	0.690721
Q_{HK}	-0.688687		

Subsequently, the present values of tax shield were calculated according to the different approaches presented in Table 1. The Fernandez (2005) model is the most complex and this calculation varies according to the nature of stochastic process of the net increase of debt. These are five cases: perpetual debt, debt of one-year maturity but perpetually rolled-over, debt is proportional to the equity value, the debt is proportional to free cash flow and the company is expected to repay the current debt without issuing new debt. Model by Couch et al. (2012) on the other hand take into account a default risk. In addition, Cooper and Nyborg (2006) described two extensions of the existing models: Modigliani & Miller (1963) and Milles and Ezzell (1985) with the assumption of growth. The analysis of the first Fernandez condition (the existence of cost of leverage) is shown in Table 4.

Table 4 Analysis of the first Fernandez condition

Model used	Company value of tax shield (EUR)	Fernandez findings	Validity in emerging markets	Author's findings
Modigliani and Miller (1963)	13 125.42	= DT	yes	-
Myers (1977)	13 125.42	= DT	yes	-
Miller (1977)	0	< DT	yes	-
Milles & Ezzell (1985)	7 414.36	< DT	yes	-
Harris & Pringle (1985)	6 990.32	< DT	yes	-
Marciniak (2003)	5 393.28	-	-	< DT
Fernandez (2005) – perpetual debt	13 125.42	= DT	yes	-
Fernandez (2005) – debt of one year maturity but perpetually rolled-over	13 125.42	= DT	yes	-
Fernandez (2005) – debt is proportional to the equity value	1 698.01	< DT	yes	-
Fernandez (2005) – the debt is proportional to free cash flow	13 125.42	= DT	yes	-
Fernandez (2005) – and the company is expected to repay the current debt without issuing new debt	13 125.42	= DT	yes	-
Cooper and Nyborg (2006) – ME with growth	-19 248.33	-	-	< MM with growth
Cooper & Nyborg (2006) – MM with growth	-8 416.13	-	-	= MM with growth
Damodaran (2006)	-11 244.80	< DT	yes	-
Practitioner´s method	-23 858.06	< DT	yes	-
Fernandez (2007) – book value of debt	-36 141.74	-	-	< MM with growth

Model used	Company value of tax shield (EUR)	Fernandez findings	Validity in emerging markets	Author's findings
Liu (2008) – earned tax shield	6 724.15	-	-	< DT
Liu (2008) – unearned tax shield	6 401.27	-	-	-
Liu (2008) – capacity of tax shield	13 125.42	-	-	= DT
Couch et al. (2012) – the first strategy	1 380.14	-	-	< DT
Couch et al. (2012) – the second strategy	1 059.40	-	-	< DT
Couch et al. (2012) – the second strategy	2 463.56	-	-	< DT
Velez-Pareja (2013)	2 971.26	-	-	< DT
Velez-Pareja (2016)	2 971.26	-	-	< DT
Velez-Pareja (2017) – discounted at cost of debt	-26 718.43	-	-	< DT
Velez-Pareja (2017) – discounted at unlevered cost of equity	-4971.35	-	-	< DT

The benchmark was the value of the tax shield by Modigliani and Miller (1963). If the value of tax shield is lower than the benchmark, the model is appropriate for perpetuity in the imperfect (real) market. These models are bold in Table 4. This condition meets eight models altogether. Cooper and Nyborg model (extended model MM) is a benchmark for models with the assumption of business growth. This condition is met by both models presented.
The second Fernandez condition (no corporation tax, cost of leverage) was tested as presented in Table 5.

Table 5 Analysis of the second Fernandez condition

Model used	Company value of tax shield (EUR)	Fernandez findings	Validity in emerging markets	Author's findings
Modigliani and Miller (1963)	0	= 0	yes	-
Myers (1977)	0	= 0	yes	-
Miller (1977)	0	= 0	yes	-
Milles and Ezzell (1985)	0	= 0	yes	-
Harris and Pringle (1985)	0	= 0	yes	-
Marciniak (2003)	0	-	-	= 0
Fernandez (2005) – perpetual debt	0	-	-	= 0
Fernandez (2005) – debt of one year maturity but perpetually rolled-over	0	-	-	= 0

Model used	Company value of tax shield (EUR)	Fernandez findings	Validity in emerging markets	Author's findings
Fernandez (2005) – debt is proportional to the equity value	**- 11 427.40**	**-**	**-**	**< 0**
Fernandez (2005) – the debt is proportional to free cash flow	0	-	-	= 0
Fernandez (2005) – and the company is expected to repay the current debt without issuing new debt	0	-	-	= 0
Cooper and Nyborg (2006) – ME with growth	0	-	-	= 0
Cooper and Nyborg (2006) – MM with growth	0	-	-	= 0
Damodaran (2006)	**-30 848.40**	**< 0**	**yes**	**-**
Practitioner´s method	**-30 848.40**	**< 0**	**yes**	**-**
Fernandez (2007) – book value of debt	0	-	-	= 0
Liu (2008) – earned tax shield	0	-	-	= 0
Liu (2008) – unearned tax shield	0	-	-	-
Liu (2008) – capacity of tax shield	0	-	-	-
Couch et al. (2012) – the first strategy	0	-	-	= 0
Couch et al. (2012) – the second strategy	0	-	-	= 0
Couch et al. (2012) – the second strategy	0	-	-	= 0
Velez-Pareja (2013)	0	-	-	= 0
Velez-Pareja (2016)	0	-	-	= 0
Velez-Pareja (2017) – discounted at cost of debt	**-40 313.4**	**-**	**-**	**< 0**
Velez-Pareja (2017) – discounted at unlevered cost of equity	**-4 971.35**	**-**	**-**	**< 0**

According to Table 5, the second condition only meets five models; they are marked in bold. On the contrary, the condition of cost of leverage meets fifteen models. Overall, the analysis shows that both conditions were met only in five cases: Fernandez (2005) for proportional value of debt to equity, Damodaran model, Practioner's method and Velez-Pareja (2017) models of net present value of tax shield. In theory, just these models are suitable for the valuation of the present value of tax shield.

Nevertheless, these conclusions are influenced by the Fernandez definition of the value of the tax shield, which has been questioned, Cooper and Nyborg (2006). For this reason, the conclusions of the analysis have been reassessed. It can be concluded that models that quantify the present value of tax shield as the product of the debt and the corporate tax rate are appropriate under the assumption of relatively stable economic conditions in the reference period. The second prerequisite for application is a short assessment period, for example, for the valuation of short to medium-term investment projects. By contrast, models that assume the value of a tax shield lower than the product of the debt and the corporate tax rate include risk. They are therefore a more appropriate alternative for long-term investment projects. In this context, we can highlight the Velez-Pareja (2013) and (2016) model, which has the lowest perpetual value, but has taken into account both the types of risk, i.e. the operational risk and the risk of bankruptcy. The only problematic point is the weighting of both types of cost of capital, as the author himself offers several ways of quantification. In addition to Velez-Pareja (2013), (2016) models, model by Couch et al. (2012) is a suitable alternative for tax shield valuation, as it takes into account default risk as well as possible changes in debt policy in the future. However, this model is not yet sufficiently used because simpler models are preferred.

One way to determine which model is appropriate from the default perspective is to use a bankruptcy prediction model (Kliestik et al., 2015) or another applicable metric is VaR (Adamko et al., 2015). On the other hand, operational risk is directly related to the efficiency of the operating process (Evans, 2018).

In addition to the previous criteria, it is possible to evaluate tax shield models according to other factors. The first is the issue of cost of capital that are subject to the Circularity problem. One of the options is quantification of the target market capital structure through the iterative process. This method is described, for example, in Marik (2018). In practice, non-traded companies quantified WACC as well as levered cost of equity on the basis of book values. As Marik and Marikova (2015) points out, this method is easier, but it creates errors in valuation. The models examined in this paper may be considered suitable for businesses in emerging markets because they are not subject to the circularity problem. Only Marciniak model is burdened with this error if levered cost of equity is estimated based on book values. On the other hand, this model also includes a systematic risk and it is therefore appropriate to remove the error caused by the circularity problem when valuing a business (investment project).

3. CONCLUSIONS

In this paper, we analyzed the impact of risk on the value of the tax shield. Based on the comparison and analysis of more than a dozen models, we can say that the tax shield is the subject of operational risk, debt, risk of market cost of debt, corporate default risk and default risk in taxes. These risks and their combinations correspond seven types of discount factor: risk-free interest rate, the cost of debt, levered cost of equity, cost of unlevered equity, stochastic discount factor, the discount factor for the debt risk and operational risk and valuations based on options.

Fernandez (2006) procedure was used to determine the impact of risk on the value of tax shield. It was found that only five models met both conditions of the given algorithm: Fernandez (2005) for proportional value of debt to equity, Damodaran model, Practioner's method and Velez-Pareja (2017) models of net present value of tax shield. These results indicate that there are only a few models in the context of corporate and / or personnel taxation that should be used. On the other hand, we revealed a lack of this methodology in terms of zero taxation. Therefore, we determined that based on the first condition of Fernandez's methodology, we can divide the models into two groups: those that are appropriate for short-term investment projects and / or stable economic conditions and these one that are appropriate for long-term risk investment projects (business valuation).

In view of the division of businesses into traded and non-traded, all models of quantifying the present value of tax shield that have been mentioned in the paper can be used. The unknown market value can be replaced by the book value. However, it should be noted that there is a risk that the tax shield will be over or underestimated. An error assessment known as the Circularity problem can be eliminated by an iterative process and estimate the cost of capital. Another option is to use a different dcf valuation method as proposed by Cooper and Nyborg (2006). As Copeland et al. (2000, p. 309) mentioned: *We leave it to the reader's judgment to decide which approach best fits his or her situation.*

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REFERENCES

Adamko, P., Spuchlakova, E., & Valaskova, K. (2015). The history and ideas behind VaR. *Procedia Economics and Finance*, *24*, 18-24.

Arzac, E.R. & L.R. Glosten (2005). A Reconsideration of Tax Shield Valuation. *European Financial Management*, *11*(4), 454-461.

Brealey, R.A., Myers, S.C., & Allen, F. (2010). *Principles of Corporate Finance*, 10th ed. New York: McGraw-Hill/Irwin.

Clememte-Almendros, J. & Sogorb-Mira, (2017). How much do the tax benefits of debt add to firm value? Evidence from spanish listed firms. *Revista de Economía Aplicada*, *25*(74), 105-129.

Couch, R. Dothan, M., & Wu, W. (2012). Interest tax shields: A barrier Options Approach. *Review of quantitative finance and accounting*, *39*(1), 123-146.

Cooper, I.A. & Nyborg K.G. (2006). The value of tax shields IS equal to the present value of tax shields. *Journal of Financial Economics*, *81*(1), 215-225.

Copeland, T.E., Koller T., & Murrin, J. (2000). *Valuation: Measuring and Managing the Value of Companies*, 3rded. New York: John Wiley& Sons.

Damodaran, A. (2006). *Damodaran on Valuation: Security Analysis for Investment and Corporate Finance*. 2nd ed. New York: John Wiley & Sons.

Doidge, D. & Dyck, A. (2015). Taxes and corporate policies: Evidence from a quasi-natural experiment. *The Journal of Finance*, *70* (1), 45-89,

Emerling, I., & Wojcik-Jurkiewicz, M. (2018). The risk associated with the replacement of traditional budget with performance budgeting in the public finance sector management. *Ekonomicko-manazerske spektrum*, *12*(1), 55-63.

Eurostat (2018). *Business demography main variables - NACE Rev. 2*. Retrieved on October 9, 2018, from: http://ec.europa.eu/eurostat/tgm/table.do?tab=table&init=1&language=en&pcode=tin00170&plugin=1.

Evans, O. (2018), Improved financial performance without improved operational efficiency: The case of Nigerian firms. *Forum Scientiae Oeconomia*, *6*(3), 25-39.

Fernandez, P. (2004a). The value of tax shield is NOT equal to the present value of tax shields. *Journal of Financial Economics*, *73*, 145-165.

Fernandez, P. (2004b). The value of tax shields and the risk of the net increase of debt (Working Paper No. 544). *SSRN Electronic Journal, February* 2004, Retrieved on October 10, 2018. 20 p. from: https://ssrn.com/abstract=506005.

Fernandez, P. (2005). The value of tax shields is not equal to the present value of tax shields: A correction (Working Paper No. 581). *SSRN Electronic Journal, January* 2005, Retrieved on October 10, 2018, from: https://ssrn.com/abstract=651206.

Fernandez, P. (2006). The correct value of tax shields: an analysis of 23 theories. (Working Paper No. 628), *SSRN Electronic Journal, June* 2006. 23 p. Retrieved October 10, 2018, from: https://www.iese.edu/research/pdfs/DI-0628-E.pdf.

Fernandez, P. (2007). A more realistic valuation: Adjusted Present Value and WACC with constant book leverage ratio. *Journal of Applied Finance, 17*(2), 13-20.

Graham, J. R. (2000). How big are the tax benefits of debt? *The Journal of Finance*, *55*, 1901-1941.

Grinblatt, M. & Liu, J. (2008). Debt policy, corporate taxes, and discount rates- *Journal of Economic Theory*. 141(1), 225-254.

Hanappi, H. (2018). Metamorphosis in Political Economy: A New Combination of Three Disparate Ideas. *Journal of Self-Governance and Management Economics 6*(2), 92–112.

Harris, R.S. & Pringle, J.J. (1985). Risk-adjusted discount rates – extensions from the average-risk case. *The Journal of Financial Research*, *8*(3), 237-244.

Investopedia (2018). Tax shield. In *Investopedia*. Retrieved 7 November, 2018, from: http://www.investopedia.com/terms/t/taxshield.asp

Kaplan, S.N. & Ruback, R.S. (1995). The Valuation of Cash Flow Forecasts: An Empirical Analysis. *Journal of Finance, 50*(4),1059-1093.

Kemsley, D. & Nissim, D. (2002). Valuation of the debt tax shield. *The Journal of Finance*, *57*(5), 2045-2073.

Kliestik T., Kocisova K., & Misankova, M. (2015). Logit and probit model used for prediction of financial health of company. *Procedia - Economics and finance*, *23*, 850-855.

Ko, J. K. & Yoon, S. (2011). Tax benefits of debt and debt financing in Korea. *Asia-Pacific Journal of Financial Studies*, *40*, 824-855.

Korteweg, A. (2010). The net benefits to leverage. *The Journal of Finance*, *65,*

Liu, Y. (2009). The slicing approach to valuing tax shields. *Journal of Banking & Finance*, *33*, 1069-1078.

Marciniak, Z. (2003). Wykorzystanie zróżnicowanych stóp kosztu kapitału do wyceny przedsiębiorstw i oceny projektów inwestycyjnych. *Proceedings of Efekty w nośćzastosowań systemów informatycznych 2003.* Warszawa, Poland, pp. 63-92.

Marik, M. & Marikova, P. (2015). Kapitálová struktura - problém výnosového ocenění podniku. *Ekonomický časopis, 63*(3), 259-277.

Marik, M. (2018). *Metody ocenovani podniku pro pokrocile*, Praha: Ekopress.

Miles, J.A. & Ezzell, J.R. (1985). Reformulating tax shield valuation: A note. *Journal of Finance, 40*, 1485-1492.

Miller, M.H. (1977). Debt and Taxes. *The journal of finance*. *32*(2), 261-275.

Modigliani F. & Miller M.H. (1963). Corporate income taxes and the cost of capital. A correction. *The American Economic Review*, *53*(3), 433-443.

Molnar, P. and Nyborg, K.G. (2011). Tax□adjusted Discount Rates: a General Formula under Constant Leverage Ratios, *European Financial Management. 19*(3), 1-10.

Myers, S.C. (1974). Interactions of corporate financing and investment decisions - Implications for capital budgeting. *Journal of Finance*, *29*(1), 1-25.

Olah, J., Karmazin, G., Mate, D., Grabara, J.K., & Popp, J. (2017). The effect of acquisition moves on income, pre-tax profits and future strategy of logistics firms. *Journal of International Studies*, *10*(4), 233-245.

Ruback, R.S. (2002). Capital cash flows: A simple approach to valuing risky cash flows. *Financial Management*, *31*(2), 85-103.

Valaskova, K., Spuchlakova, E., & Adamko, P. (2015). Non-parametric bootstrap method in risk management. *Procedia Economics and Finance*, *24,* 701-709.

Van Binsbergen, J., Graham, J., & Yang, J. (2010). The cost of debt. *The Journal of Finance*, *65*, 2089-2136.

Velez - Pareja, I. (2013). Return to Basics: Are You Properly Calculating Tax Shields? *Análisis Financiero*. *122*, 6-17.

Velez-Pareja (2017). Do Personal Taxes Destroy Tax Shields? A Critique to Miller's (1977) Proposal. *Financial Reforms in Emerging Market Economies*, *53*(10), 2199-2214.

Velez-Pareja, I. (2010). Risky Tax Shields: An Exploratory Study. *Cuadernos de Administración, 23*(41), 213-235.

Velez-Pareja, I. (2016). Tax shield, financial expenses and losses carried forward. *Cuadernos de Economica*, *35*(69), 663-689.

Weissova, I., Kollar, B., &Siekelova, A. (2015). Rating as a useful tool for credit risk management. *Procedia Economics and Finance*, *26*, 278-285.