ANALYSIS OF A NON-LINEAR DYNAMIC FINANCIAL SYSTEM

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ABSTRACT

Some of the major trends of the present include the study of various model situations, orientation in simulated conditions, searching for starting points or optimum solutions, etc.; but in modelling complex relationships within economic systems, it is necessary to take into account the time delay in situations where the dynamic behaviour of the model also depends on its previous states. In this article, the authors ana-lyse the dynamic model expressed by a set of non-linear ordinary differential equations with delayed arguments describing the financial system.

The aim of the authors is to analyse the behaviour of the dynamic model of the financial system in terms of its solvability and stability in the event that the model takes into account the impact of the history of the demand for investments. It has been proven that under the assumptions given in the article, our task has only one solution, and this solution is continuously differentiable at the interval being investigated. Furthermore, the stability conditions were formulated and a numerical solution for differently set system parameters presented.

The authors have demonstrated that the system has a complex dynamic behaviour that is significant-ly affected by the length of the delay of the response to the change in demand for investments; and this is the factor that influences the stability of the system. The results are demonstrated on a specific example, and the behaviour of the model is presented by computer simulation; the Maple system is used to graphically represent the results.

KEYWORDS: *asymptotic stability, differential equations, financial system, modelling, simulation*

1. INTRODUCTION

The natural trend of most scientific disciplines is to investigate various model situations that subsequently allow us to analyze simulated processes, or more precisely – to specify the conditions under which they work, to derive practical conclusions from the findings made, or to find an optimum solution. If an economist wants to assess the impacts of certain real steps in the economic system, he or she can create an economic model and use historical data to verify his or her theory. Very often is it thus necessary for him or her to deal with the modelling of complex economic phenomena and systems, analysis and verification of these models, prediction and optimal decision making.

The theory of non-linear dynamics of the systems is then able to show the relationship of the whole to the change of its individual parts, and what the differences are of the whole and its parts. The study of non-linear systems was spread especially by mathematicians, such as Ljapunov, Pontryagin, Andronov. Their work was then followed by others, such as Smale, Kolmogorov,

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Arnol'd or Sinai, and in 1975, a new kind of movement in dynamic systems, now called a chaotic movement, was discovered.

These findings can be used in many areas of economics; they can be applied, for example, to disasters, the economic cycle, economic growth, economic chaos, the impact of stochastic changes on socio-economic structures, rapid and slow socio-economic processes, or the relationship between microeconomic and macroeconomic structures.

The development of the chaos theory and its possible application in economics led to the discovery of possible randomises contained in the macroeconomic decisions of governments, which can very hardly be prevented, for example, by a change in monetary policy. In the areas of finance or social economy, internal structures are often non-linear and interrelationships very complex. A non-linear dynamic model can explain the functioning of the financial system that is part of the economic system, although it is also necessary to take into account that a certain variable depends on its previous values or on previous values of other variables.

In the introductory chapters of this article, the current state of the issue of non-linear systems in the field of economics is briefly described and a dynamic non-linear financial system defined. This introductory part serves as a basis for formulating other relationships. The aim of the article is to investigate, using the means of mathematical analysis, the behaviour of the dynamic model of a financial system in terms of its solvability and stability in the event that the model takes into account the impact of the history of demand for investments on the behaviour of the system. Using a modern theory of functional differential equations and suitable software, numerical solutions to specific tasks can be then simulated.

2. LITERATURE REVIEW

In many areas of science, the theory of chaos has received great attention in recent decades. The first chaotic attractor was found in the weather forecast model and is known as a Lorenz chaotic system (see, for example (Lorenz, 2004)). Later studies have demonstrated the existence of chaos, for example, in the chemical reaction model (Rossler, 1976) or in the electrical circuit systems (Chua et al., 1986). Since 1989, when Hüber published his first treatise on chaos management, chaos management has attracted great attention thanks to its potential applications in physics, chemical reactors, control theory, biological networks, artificial neural networks, telecommunications, etc. Chaos is also widely used in digital communication due to its complex dynamic behaviour (Chen & Ueta, 1999). The first three mentioned systems belong to the group of generalized Lorenz systems. They are currently also being joined by multi-wing and multi-scroll chaotic systems (Yu et al., 2011; Xu, 2014). In addition to classical chaotic systems described by systems of ordinary first-order differential equations, fractional chaotic systems (Xu et al., 2015) are also studied.

In recent decades, the chaos control theory has been devoted a major effort, particularly in the field of stabilization of unstable equilibrium points and unstable periodic solutions (Hubler, 1989). Methods have been developed especially for chaos suppression in various chaotic systems (He & Oppewal, 2017; Chen et al., 2014).

In the areas of finance, internal structures are often non-linear and interrelationships very complex, so the studies examining the effects of the internal structural characteristics of such a system represent a system as a set of differential equations with possible chaotic behaviour. Chaos in the financial system was first demonstrated in 2001 (Ma & Chen, 2001). Chen (Chen et al., 2014) and Yu, Cai and Li (Yu et al., 2011) introduced a 4D chaotic financial system in their articles and achieved control over it by linear feedback and a speed feedback controller. Llibre and Valls (2018) studied the global dynamics of the financial model. Since many financial systems have memory effects, the systems with memories are also examined by means of fractional-order differential equation systems (Li et al., 2017). A similar topic is also dealt with by the authors (Xu et al., 2018).

The phenomena related to the memory of economic and financial systems are examined by the authors (Tacha et al., 2018).

3. NON-LINEAR DYNAMIC FINANCIAL SYSTEM

The financial system can be understood as a set of markets, institutions, laws, regulations and techniques through which financial transactions of all kinds are realized. Models of dynamic financial systems may include some trends pointing out the importance of sensitivity of parameters to the initial values, from which the systems then evolve to structural changes. These can cause fluctuations over a long period of time. However, it should be noted that the occurrence of these characteristics may not be necessarily caused solely by the activities of the economy. Similar characteristics can then change into mechanisms that are capable of provoking a change in the structure of the economy to a state in which chaotic behaviour occurs (Ma & Chen, 2001)
In publications (Ma & Chen, 2001), the key part of the financial model was simplified for the sake of simplification. Based on a thorough analysis and experiments, it was then decided that the model would include the following variables: x as the interest rate, y as the demand for investments, and z as the price index.
Interest rate x is influenced by the factors from two aspects. Firstly, the investment market, where the interest rate is influenced by the opposite; for example, the surplus of investments over savings will be reflected in a lower interest rate. The second aspect includes the structural changes based on the prices of the goods.
The magnitude of the changes in demand for investments y is in relation to the investment rate, and at the same time in the opposite proportion to the investment cost and the interest rate. Return on investments is assumed to be constant over a certain period of time.
Changes in the z price index are influenced by the difference in demand and supply of trading markets, and are also affected by the size of inflation. In this model, it is assumed that the size of supply and demand does not change over a certain period of time, and at the same time that the total amount of demand and supply is in the reverse proportion to prices. And also, that the change in the magnitude of inflation can be expressed by a change in the real interest rate, and that the magnitude of inflation is equal to the difference between nominal and real interest rates.
A very important characteristics of these variables is their sensitivity to the change of known information in the economy. The question of the influence of the sensitivity of variables on the change of information is the problem being investigated, and for this reason the changes of these variables in time are defined as state variables: $\dot{x} = dx/dt, \dot{y} = dy/dt, \dot{z} = dz/dt$.
The model of the financial system is then represented by three differential equations:

$$\begin{aligned} \dot{x} &= z(t) + (y(t) - a)x(t) \\ \dot{y} &= 1 - by(t) - x^2(t) \\ \dot{z} &= -x(t) - cz(t), \end{aligned} \tag{1}$$

where a represents household savings, b investment costs and c the elasticity of demand of commercial markets. We assume that all three model parameters are positive.
These three partial equations composed into one set are the basis of the dynamic financial model for x, y and z, including the mutual influence of the information. The model obtained in this way contains independent parameters that affect the system in question, but in this case the exact values of the individual parameters are not that important; more important are their mutual relationships and the resulting changes in the behaviour of the system.
The dynamic financial system, represented by a set of ordinary differential equations, should contain – according to (Ma & Chen, 2001) – chaotic behaviour. This statement is examined by the

numerical experiments that are conducted with this system and confirmed also in other publications (Chen et al., 2014).
The effects of time delay in non-linear systems have been investigated by various authors for many years (Chen, 2008; Yan & Chu, 2006; Novotná & Štěpánková, 2015). The time delay in real financial systems has been repeatedly confirmed. In our paper, we will deal with a time delay relating to the demand for investments y. By adding feedback to the second equation of the system (1), we get the system:

$$\begin{gathered}\dot{x} = z(t) + (y(t) - a)x(t) \\ \dot{y} = 1 - by(t) - x^2(t) + k[y(t) - y(t-\tau)] \\ \dot{z} = -x(t) - cz(t),\end{gathered} \tag{2}$$

where $\tau > 0$ is the length of time delay and k the power of feedback.

4. MODEL ANALYSIS

If the linear system is stable, it is stable under all initial conditions. The behaviour of the non-linear system is much more complicated. Non-linear systems may be stable under certain initial conditions and unstable under other initial conditions.
Ljapun stability of equilibrium states can be investigated for non-linear system (2) by linearizing its equation in the vicinity of each equilibrium state and determining the stability of the substitute linear system. Then the non-linear system behaves in the vicinity of the equilibrium state (i.e. the singular point) similarly to the linearized system. The investigation of stability of the linearized system also applies to the non-linear system. But only in the vicinity of the singular point.
To investigate the local stability of system (2) in the vicinity of equilibrium points, we will perform the task of finding the equilibrium for system (1).
Equilibrium can be easily determined $= \left(0, \frac{1}{b}, 0\right)$.
It is also true that equilibrium is stable if the following applies $c - b - abc < 0$ and if we use the principle of linearized stability, we can make decisions using a non-linearized system around equilibrium E.

$$\begin{gathered}\dot{x} = z(t) + \left(\frac{1}{b} - a\right) x(t) \\ \dot{y} = -by(t) + k[y(t) - y(t-\tau)] \\ \dot{z} = -x(t) - cz(t),\end{gathered} \tag{3}$$

For further steps we define that $\tau_i = \frac{1}{\sqrt{2kb-b^2}} arccos\frac{k-b}{k} + \frac{2j\pi}{\sqrt{2kb-b^2}}$.
If we use sentence 1 from (Gao & Ma, 2009) and anticipate that coefficients a, b, c meet the following assumptions:
$c - b - abc < c \ \wedge \ c + a - \frac{1}{b} > 0$, or
$\left(c + a - \frac{1}{b}\right)^2 - 4(c - b - abc) > 0$,
then the solution of system (2) can be divided into the following cases depending on the length of delay τ:

1. $\tau \in \langle 0, \tau_0)$ - the system is asymptotically stable
2. $\tau > \tau_0$ - the system is asymptotically unstable

3. $\tau = \tau_j,\ j = 0,1,2,\dots$ - the Hopf bifurcation occurs in the system.

We will add the "historical function" and an initial condition to the system of differential equations with delay (2) being solved.

We will be dealing with the following task

$$\begin{gathered}\dot{x} = z(t) + \left(\frac{1}{b} - a\right)x(t)\\ \dot{y} = -by(t) + k[y(t) - y(t-\tau)]\\ \dot{z} = -x(t) - cz(t),\\ (x(0), y(0), z(0)) = (x_0, y_0, z_0),\ t \in [0, T]\end{gathered} \tag{4}$$

where $y(t) = y_h(t)$ for $t < 0$. We should add that $y_h(t)$ is the so-called "historical function", $y_0 = y_h(0)$ and x_0, z_0 are positive constants.

Let us now mark

$$\chi_{[0,T]}(t) = \begin{cases}1 & t \geq 0\\ 0 & t < 0\end{cases} \tag{5}$$

It is probably possible – using the above marking – to write task (4) in the form of

$$\begin{gathered}\dot{x} = z(t) + \left(\frac{1}{b} - a\right)x(t)\\ \dot{y} = -by(t) + k\left[y(t) - (\chi_{[0,T]}(t-\tau)y(t-\tau) + (1 - (\chi_{[0,T]}(t-\tau))y_h(t-\tau)\,)\right]\\ \dot{z} = -x(t) - cz(t),\\ (x(0), y(0), z(0)) = (x_0, y_h(0), z_0),\ t \in [0, T]\end{gathered} \tag{6}$$

It means that task (6) is equivalent to task (4) (see (Kiguradze, 1988; Kiguradze, 1997; Kiguradze & Půža, 2003; Kiguradze & Půža, 1997; Bobalová & Maňásek, 2007); the above literature on fully general linear marginal tasks for functional differential equations shows that under these assumptions, our task (4) has only one solution, which is continuously differentiable in interval $[0, T]$. If functions $x(t), y(t), z(t)$ and $y_h(t)$ had discontinuities of the jump type in the corresponding intervals, the solution of task (6) will remain continuous, but will not have a derivative in the corresponding points.

Now we will look for the solution of our task (6) by the method of successive approximations:

- we choose randomly functions $x_0(t), y_0(t), z_0(t)$ continuous in interval [0,T],
- we gradually calculate the n-th approximation of the solution sought, $n \in N$, using the auxiliary tasks

$$\begin{gathered}\dot{x}_n = z_n(t) + \left(\frac{1}{b} - a\right)x_n(t)\\ \dot{y}_n = -by_n(t) + k\left[y_n(t) - (\chi_{[0,T]}(t-\tau)y_{n-1}(t-\tau) + (1 - (\chi_{[0,T]}(t-\tau))y_h(t-\tau)\,)\right]\\ \dot{z}_n = -x_n(t) - cz_n(t),\\ (x_n(0), y_n(0), z_n(0)) = (x_0, y_h(0), z_0),\ t \in [0, T]\end{gathered} \tag{7}$$

Based on the continuity of the right sides of the equations of system (6) and continuity of the "initial" functions, it can be stated that tasks (7) are unambiguously solvable for each $n \in \mathrm{N}$ and that a uniform convergence of the constructed sequence of the solution of tasks (7) to solutions of system (6) follows from the fulfilment of the conditions of unambiguous solvability of the initial task for system (6).

5. NUMERICAL EXPERIMENT

Because of the fulfilment of the convergence conditions, the method of successive approximations is used for the numerical solution.

5.1 Software used

We can no longer imagine solving modern scientific problems without the use of computing technology. However, the computer itself is not enough. It is necessary to equip it with adequate software and an employee who properly processes the problem. Primarily for scientific purposes and numerical solutions, an imperative programming language, FORTRAN, was developed in 1954. Although this programming language was revolutionary in its time, with the development of time and computing technology (especially with the onset of the PC platform), which became available to the general public, FORTRAN became more of an obstacle to development.
In 1963, an Utrecht University Professor and later laureate of Nobel Prize in Physics, Martinus J. G. Veltman, created the Schoonship system – the first CAS system for mathematical and physical calculations. It was followed by the systems Reduce (since 1968) and Macsyma (since 1970), which are still commercially available and have been programmed in the LISP language. The free version of the Macsyma system, the so-called Maxima2, has been developed since 1998. The systems Maple (beginning in 1981) and Mathematica (since 1988) now dominate the market. Smaller competitors include the Axiom5, MuPAD and MathCAD systems that do not have such a wide range of applications. Another known system is Derive6 (since 1988), which is focused on simplicity of control and is usually only used in teaching.
In our article, we will use the Maple system developed at Waterloo University in Canada to simplify and accelerate mathematical computations. Unlike classical programs for numerical computations (e.g. MATLAB – also includes a symbolic calculation tool), it models mathematical operations with symbolic expressions. Maple enables you to perform both symbolic and numeric calculations, so you can create function charts or program your own functions or procedures.
To solve differential equations and their sets, the dsolve command is used in Maple.
In general, we can say that Maple works with different ways of solving differential equations:
1. An analytical solution where Maple looks for a solution in the form of a function/functions;
2. A solution using endless rows where Maple searches for a solution expressed in the form of an endless power series;
3. Numerical and closely related graphic solutions.
All the solution types can be invoked in Maple using the mentioned dsolve command. This command is a very effective and popular tool for solving differential equations. The obligatory parameter of the dsolve procedure is a differential equation (if only the expression is given, it is considered as equal to zero). The diff procedure is used to express the derivative. When entering, a user can select a large variety of parameters, such as the initial conditions for determining integration constants. The initial conditions, together with the equation, are enclosed in set brackets. In the initial setting, this command symbolically solves the entered equation. For a numerical solution, it is necessary to indicate the numeric parameter. Detailed information about this command and all its supported entries can be found in the system manual at (Help Maplesoft, 2018).
In our article, we will limit ourselves to its basic use for solving a system of ordinary differential equations both in the general form and with initial conditions (a Cauchy initial task).

5.2 Simulation

To demonstrate the possibility of a new approach to solving the original problem, we assume that "historical development" before t = 0 can be simulated using the function $y = sin(\pi t/18)$.

In Figure 1, we see the behaviour of the system without feedback regardless of whether the condition $c - b - abc < 0 \wedge c + a - \frac{1}{b} > 0$ is met or not.

Example 1

Let us now consider situation a) as the initial state. In this case, the financial system stabilizes – as expected – at the equilibrium-based level, which, however, in practical terms means that it is no longer flexible; in addition, zero level x and z does not correspond to the economic reality. In case b) we see a situation where the level of savings has declined, the system is able to respond to stimuli and the solution found is to be periodic. However, if the level of savings is still declining, the system is no longer stable, and chaotic behaviour is gradually taking place, as we can see in case c).

Example 2

Further simulations will be made on the assumption that the system would be in a stable state if feedback in the case of the demand on investments was not taken into account. The value of the parameters corresponds to the situation when the system is in a stable state (values of the parameters $a = 10,\ b = 0.2, c = 1.6)$ and at the same time we will consider the existence of the feedback, whose power is expressed by the k parameter, whose value we set at $k = 0,2$. We will then vary the length of the time delay depending on the stability conditions 1-3 from the "Model Analysis" section.

a)	b)	c)
$c - b - abc < 0 \wedge c + a - \frac{1}{b} > 0$	$c - b - abc > 0 \wedge c + a - \frac{1}{b} > 0$	$c - b - abc > 0 \wedge c + a - \frac{1}{b} < 0$
a=15 ; c=0.5; b=0.125	a=6.5; c=0.5; b=0.125	a=1.5; c=0.5; b=0.125

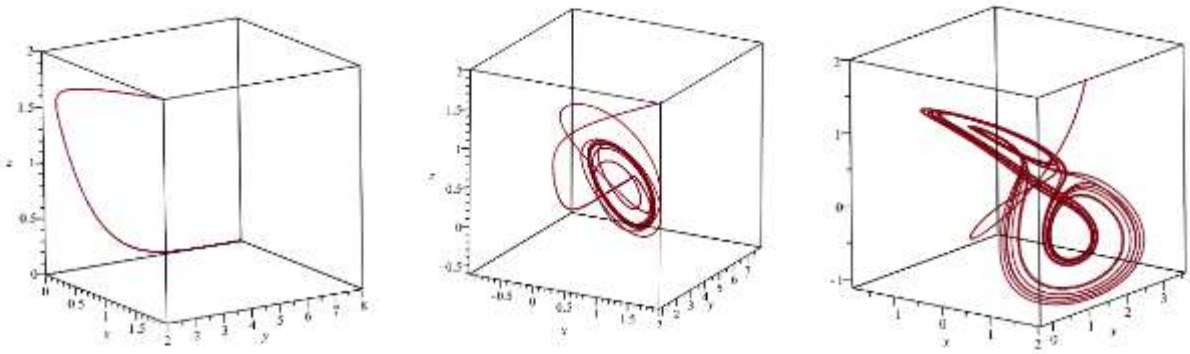

Figure 1. Example 1

Source: Our own numerical simulation

a)	b)	c)
$\tau < \tau_0$	$\tau = \tau_0$	$\tau > \tau_0$
$\tau = 6$	$\tau = 7.853981635$	$\tau = 10$

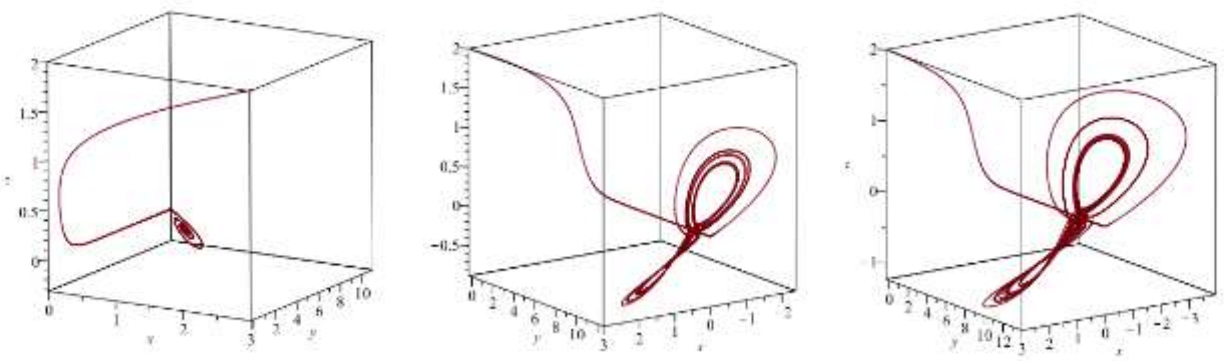

Figure 2. Example 2
Source: Our own numerical simulation

As can be seen from Figure 2, if the speed of the system's response to a change in investments slows down above a certain limit, then the initially asymptotically stable system gets into the unstable state and starts to show chaotic behaviour, and we can monitor the so-called Hopf bifurcation. In other words – as follows from the above theoretical analysis and results of numerical simulation, the behaviour of the dynamic financial system described by the set of equations (2) can show both balance and chaotic behaviour depending on the length of time delay τ. This suggests that the length of the system's response to changing demand for investments is a very sensitive factor for the stability of the entire system.

6. CONCLUSION

In this article, we focused on the analysis of a dynamic non-linear financial system with a time delay. Based on our analysis, we can conclude that the system exhibits complex dynamic behaviour, which is significantly affected by the length of the delay of the response to the change in demand for investments, and this is the factor that influences the stability of the system.
We have proven that the financial system under our review is very sensitive to changing parameters. We have also found that a stable state occurs if the price index and the interest rate are zero, which is inconsistent with normal reality. This leads to the idea that the state of financial instability is more advantageous, but on the other hand, chaotic behaviour in this case appears very easily.
If we are able to deepen our knowledge of the behaviour of individual variables of such a system when changing the values of each parameter, we will be able to better understand the fundamental changes in the economic variables of the real world and we will be able to sensitively keep individual components of the system in a state where possible shocks will not divert them too much, but at the same time, continuous adaptation to changes will be taking place.

ACKNOWLEDGMENT

This paper was supported by grant FP- S-18-5524 "Information and Knowledge Management in the age of Industry 4.0" of the Internal Grant Agency at Brno University of Technology.

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